CHAPTER 7

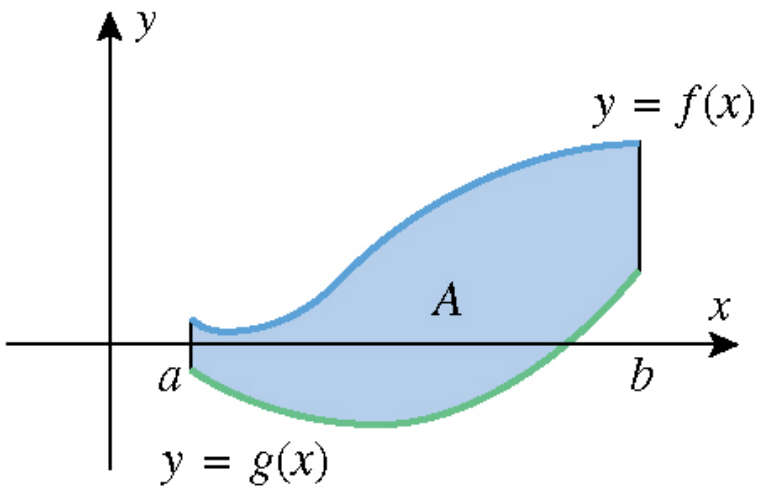
Applications of the Definite Integrals

7.1 Area Between Two Curves

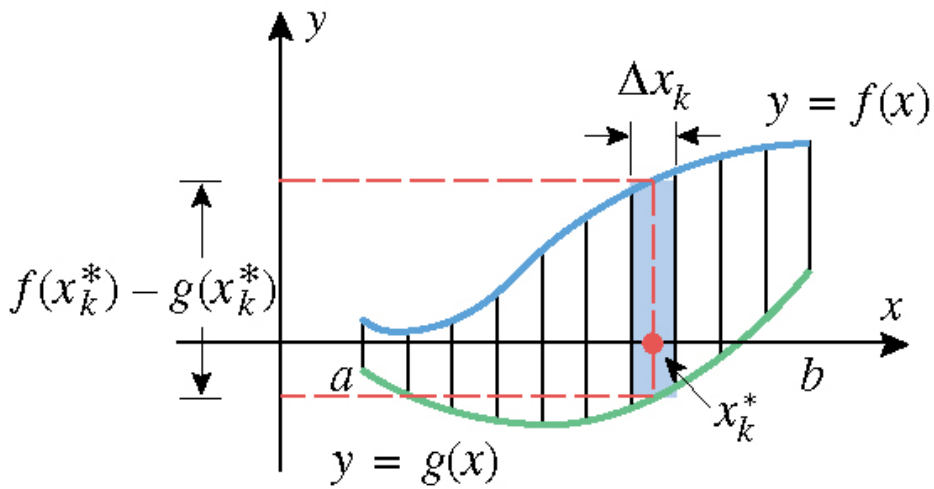
7.1.1 FIRST AREA PROBLEM. Suppose that f and g are continuous functions on an interval $[a, b]$ and

$$f(x) \geq g(x) \quad \text{for} \quad a \leq x \leq b$$

[This means that the curve $y = f(x)$ lies above the curve $y = g(x)$ and that the two can touch but not cross.] Find the area A of the region bounded above by $y = f(x)$, below by $y = g(x)$, and on the sides by the lines $x = a$ and $x = b$ (Figure 7.1.2*a*).



(*a*)



(*b*)

$$A \approx \sum_{K=1}^{n} \left[f(x_k^*) - g(x_k^*) \right] \Delta x_k$$

$$A = \lim_{\max \Delta x_k \to 0} \sum_{K=1}^{n} \left[f(x_k^*) - g(x_k^*) \right] \Delta x_k = \int_a^b \left[f(x) - g(x) \right] dx$$

7.1.2 AREA FORMULA. If f and g are continuous functions on the interval $[a, b]$, and if $f(x) \geq g(x)$ for all x in $[a, b]$, then the area of the region bounded above by $y = f(x)$, below by $y = g(x)$, on the left by the line $x = a$, and on the right by the line $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,dx \qquad (1)$$

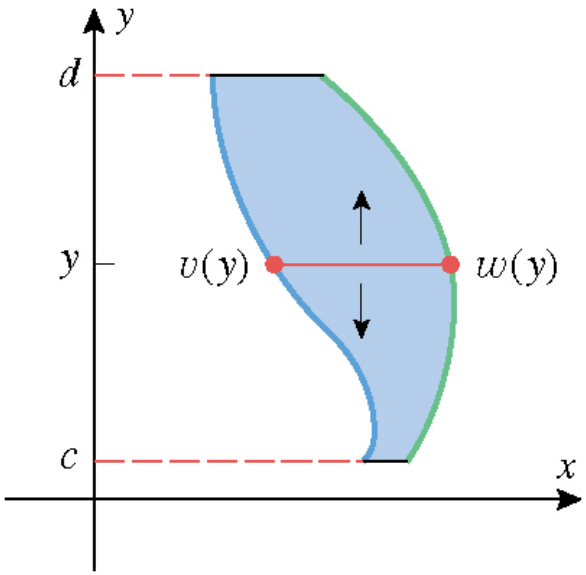
Example 1

Find the area of the region that is enclosed between the carve

$y = x^2$ *and* $y = \sqrt{x}$

Example 2

Find the area of the region enclosed by $x = y^2$ *and* $y = x - 2$



7.1.4 AREA FORMULA. If w and v are continuous functions and if $w(y) \geq v(y)$ for all y in $[c, d]$, then the area of the region bounded on the left by $x = v(y)$, on the right by $x = w(y)$, below by $y = c$, and above by $y = d$ is

$$A = \int_c^d [w(y) - v(y)]\,dy \tag{4}$$

Example 3

Solve example 2 by integrating with respect to *y.*

Example 4

Find the area of the region enclosed by

$$y = x^3,\ y = x + 6 \ \textit{and}\ \ 2y + x = 0$$