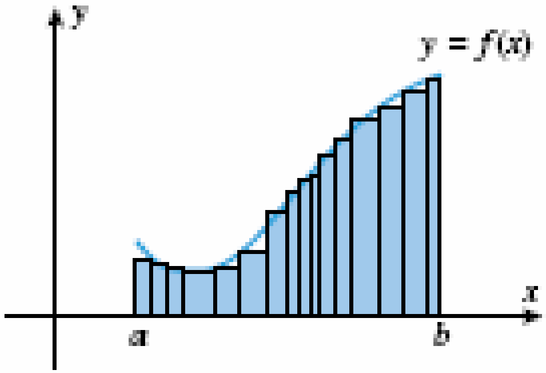
6.5 Definite Integral



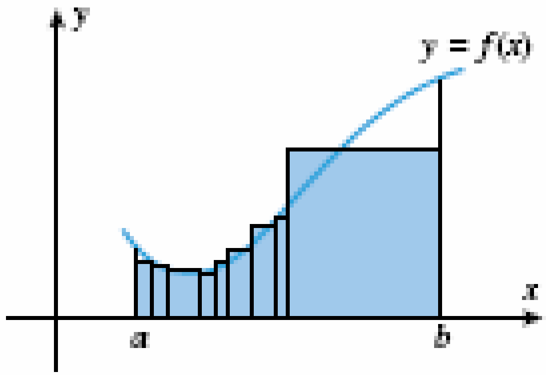


Figure 6.5.2

A *partition* of the interval $[a, b]$ is a collection of numbers

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

that divides $[a, b]$ into n subintervals of lengths

$$\Delta x_1 = x_1 - x_0, \quad \Delta x_2 = x_2 - x_1, \quad \Delta x_3 = x_3 - x_2, \ldots, \quad \Delta x_n = x_n - x_{n-1}$$

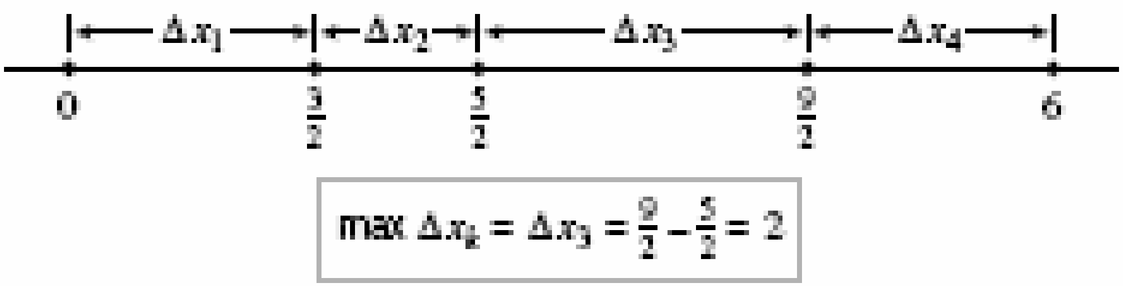


Figure 6.5.3

$$A = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*) \Delta x_k \triangle ABC$$

Definition.

A function ***f*** is said tot be **Riemann integrable** on [a, b]

If the limit

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*) \Delta x_k = \int_a^b f(x) dx$$

Exists and does not depend on the choice of the partitions or on the point x_k^*

Example 1

$$a) \int_1^3 (x+1) dx \qquad b) \int_{-2}^{2} x dx \qquad c) \int_0^2 \sqrt{4-x^2} dx$$

Properties of the definite integral

$$(a)\ \int_a^a f(x)\,dx = 0 \qquad (b)\ \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Example2

$$a)\ \int_2^2 (x^2+2)\,dx = 0 \qquad b)\ \int_1^3 x\,dx = -\int_3^1 x\,dx$$

Theorem

If ***f*** and ***g*** are integrable on [*a, b*] and if ***c*** is a constant , then

cf , ***f + g*** and ***f - g*** are integrable on [*a, b*] and

$$(a)\ \int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$(b)\ \int_a^b [f(x) \mp g(x)]dx = \int_a^b f(x)dx \mp \int_a^b g(x)dx$$

Theorem

If ***f*** is integrable on a closed interval containing the three points *A, b, and c, then*

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Theorem

(a) If ***f*** is integrable on [*a, b*] and $f(x) \geq 0$ on $[a,b]$ for all *x* in [*a, b*], then

$$\int_a^b f(x)dx \geq 0$$

(b) If ***f*** and ***g*** are integrable on [*a, b*] and $f(x) \geq g(x)$ For all *x* in [*a, b*], then

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx$$

Example 3

(a) $\int_{-2}^{3} (x^2+5)dx \geq 0 \quad \text{since } x^2+5 \geq 0, \text{ for all } x \in [-2,3]$

(b) $\int_{3}^{7} (2x^3+2)dx \geq \int_{3}^{7} (2x^3-1)dx$

(c) $\int_{-2}^{3} f(x)dx - \int_{5}^{3} f(x)dx = \int_{-2}^{5} f(x)dx$

(d) Exercise 15

(e) See example 4 p.392

(f) Exercise

Discontinuities and Integrablitiy

Definition

f is bounded on interval ***I*** if there exist $M \geq 0$ Such that

$$|f(x)| \leq M$$

6.5.8 THEOREM. *Let f be a function that is defined on the finite closed interval $[a, b]$.*

(a) *If f has finitely many discontinuities in $[a, b]$ but is bounded on $[a, b]$, then f is integrable on $[a, b]$.*

(b) *If f is not bounded on $[a, b]$, then f is not integrable on $[a, b]$.*