6.4 Sigma Notation:

Let $f(k)$ be a function of integers. The sum $f(1)+.....+f(n)$ may be written in summation on sigma notation as: $\sum_{k=1}^{n} f(k)$

Where **k** index; **k=1** starting value (lower limit)

k=n ending value (upper limit)

Remark:

1. we usually write a_k for $f(k)$

2. The letter used for index is not important $\sum_{i=1}^{5} i^2 = \sum_{k=1}^{5} k^2$

Change the Limits of Summation

$$\sum_{k=3}^{6} 2k = \sum_{k=1}^{4} 2(k+2) = \sum_{j=4}^{7} 2(K-1)$$

Example1

$$a)\ \sum_{i=1}^{4} \ln i \qquad b)\ \sum_{n=1}^{4} 7 \qquad c)\ \sum_{k=1}^{3} (-1)^k (2k+1)\sin(k\frac{\pi}{2})$$

6.4.1 Theorem

$$(a)\ \sum_{k=1}^{n} c\ a_k = c\sum_{k=1}^{m} a_k$$

$$(b)\sum_{k=1}^{n}(a_k \mp b_k) = \sum_{k=1}^{n} a_k \mp \sum_{k=1}^{n} b_k$$

3.

6.4.2 Theorem

$$(a)\ \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad (b)\ \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(c)\ \sum_{1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Example 2

$$\sum_{k=1}^{6}\left(k^3-3k-2\right)$$

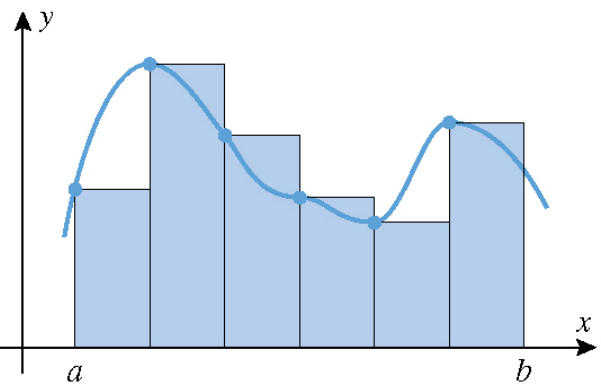
Example 3

$$(a)\ \lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2}$$

$$(b)\ \sum_{k=1}^{50}\left(\frac{1}{k}-\frac{1}{k+1}\right)$$

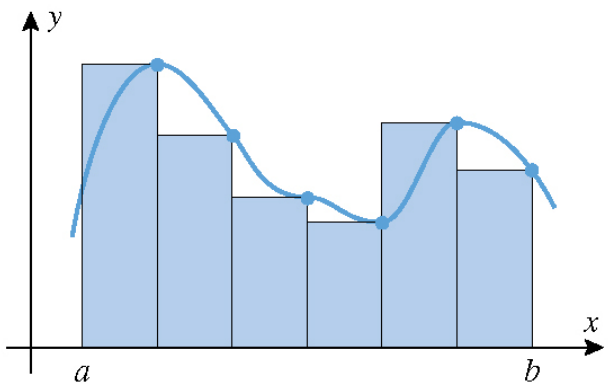
A Definition of Area

Suppose $f(x)$ continuous and nonnegative on $[a,b]$



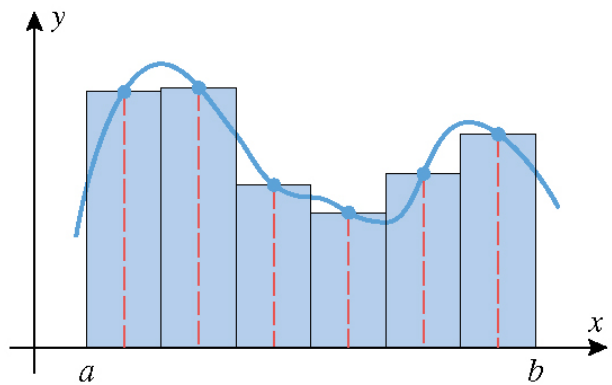
Left endpoint approximation

(a)



Right endpoint approximation

(b)



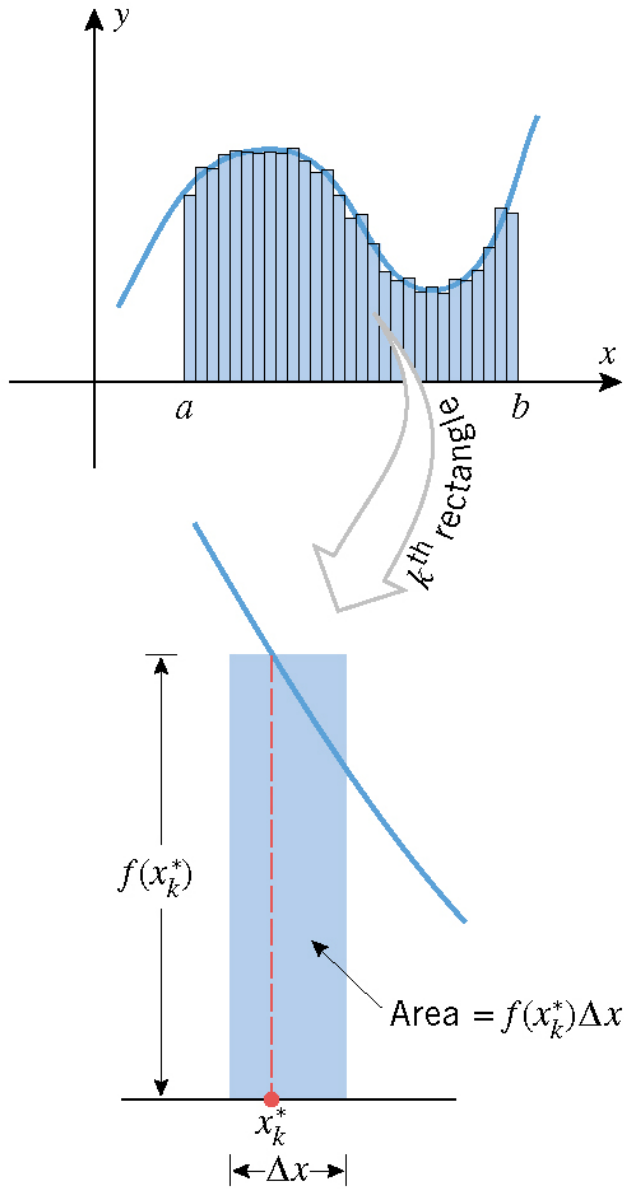
Midpoint approximation

(c)

$$\Delta x = \frac{b-a}{n}$$

$$Area = f(x_k^*)\Delta x$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$



If, as shown in Figure 6.4.6, the subinterval $[a, b]$ is divided by $x_1, x_2, x_3, \ldots, x_{n-1}$ into n equal parts each of length $\Delta x = (b - a)/n$, and if we let $x_0 = a$ and $x_n = b$, then

$$x_k = a + k\Delta x \quad \text{for } k = 0, 1, 2, \ldots, n$$

Thus,

$$x_k^* = x_{k-1} = a + (k - 1)\Delta x$$ Left endpoint

$$x_k^* = x_k = a + k\Delta x$$ Right endpoint

$$x_k^* = \tfrac{1}{2}(x_{k-1} + x_k) = a + \left(k - \tfrac{1}{2}\right)\Delta x$$ Midpoint

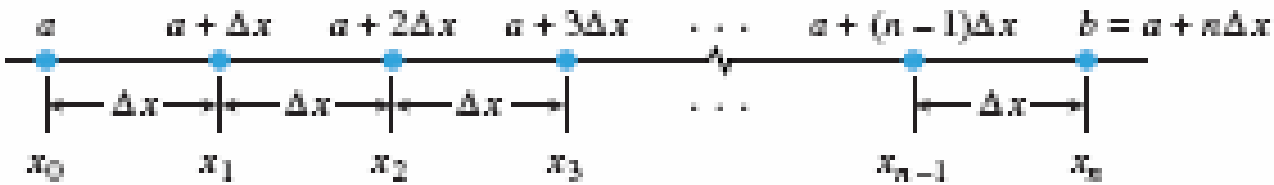


Figure 6.4.6

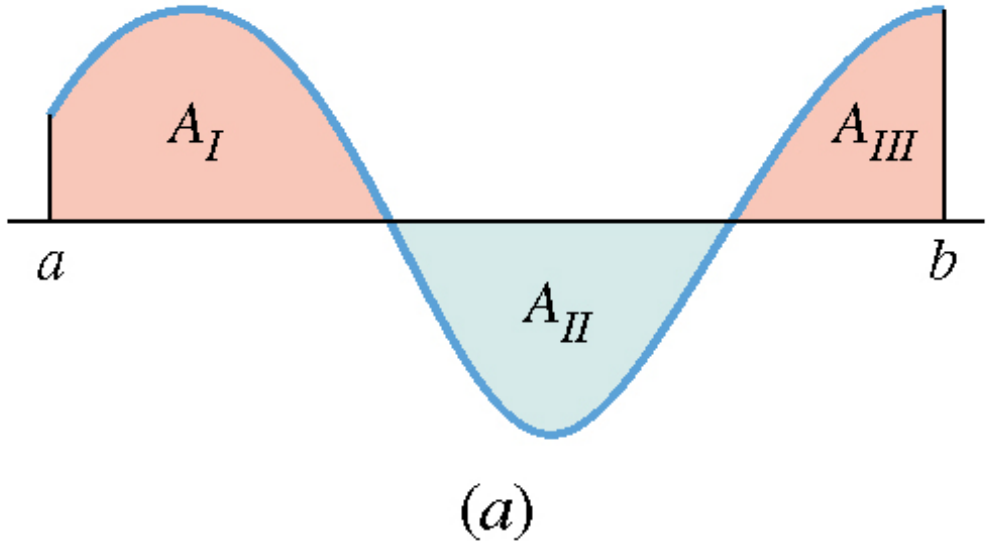
Example 4

Find the area under the graph $y = 9 - x^2$ *from* $x = 0$ *to* $x = 3$

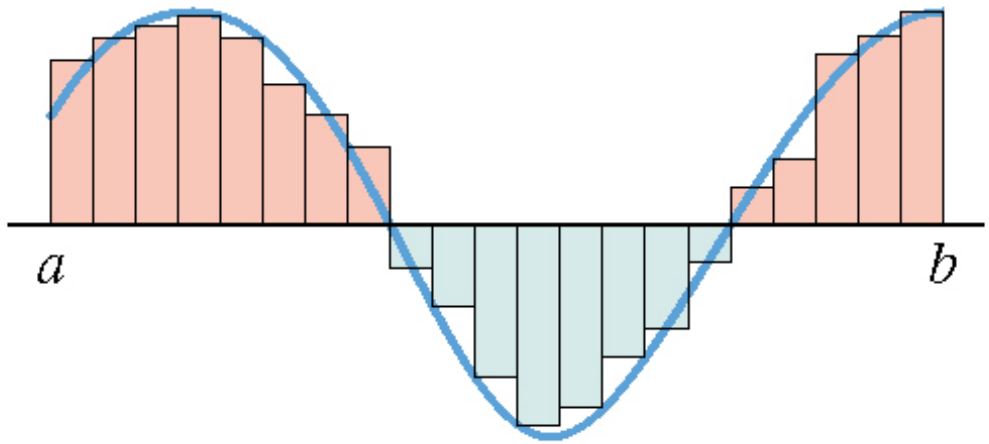
Net signed Area

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

=(AI+AIII)-(AII)



(a)



(b)