6.3 Integration by Substitution

F is an **antiderivative** of *f*

$$\frac{d}{dx}[F(g(x))] = F'(g(x)) \cdot g'(x)$$

$$\int F'(g(x))g'(x)dx = F(g(x)) + c$$

$$\int f(g(x))g'(x)dx = F(g(x)) + c$$

$$u = g(x) \Rightarrow du = g'(x)dx$$

$$\int f(u)du = F(u) + c$$

Example 1 Evaluate

$a) \int (x^3+2)^{25} 3x^2 dx \qquad b) \int \cos^3 x \sin x dx \qquad c) \int \frac{1}{\sqrt{x}} \sin \sqrt{x} dx$

$d) \int \frac{3x}{\sqrt{4x^2+5}} dx \qquad e) \int z\sqrt{z+1} dz \qquad f) \int \frac{e^x}{1+e^x} dx$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c$$

Example 2

$$\int \frac{dx}{x\sqrt{2x^2-\pi}}$$