7.2 Indefinite Integral: Integral curves

6.2.1 Definition: A function F is an antiderivative of a function f on an interval I if $F'(x)=f(x)$ for all x in the interval.

Example 1 $\frac{x^4}{4}, \frac{x^4}{4}+3, \frac{x^4}{4}-6, \frac{x^4}{4}+c$ are antiderivatives of $f(x)=x^3$

6.2.2 Theorem If $F(x)$ is an antiderivative of f(x) on an interval I, then for any constant c the function $F(x)+c$ is also antiderivative on I.

Note: The process of finding the antiderivative is called antidifferentiation or integration

$$\frac{d}{dx}[F(x)]=f(x) \Leftrightarrow \int f(x)dx=F(x)+c$$

Indefinite integral

Constant integration

$$\int x^3 dx=\frac{x^4}{4}+C \qquad \int \cos t\,dt=\sin t+c \qquad \int e^u du=e^u+c$$

Integral Formulas:

Table 6.2.1

DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \quad (r \neq -1)$	$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x\,dx = -\cos x + C$
5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$

8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$ $\quad\quad$ $\int \csc x \cot x\, dx = -\csc x + C$

9. $\frac{d}{dx}[e^x] = e^x$ $\quad\quad$ $\int e^x\, dx = e^x + C$

10. $\frac{d}{dx}\left[\frac{b^x}{\ln b}\right] = b^x \quad (0 < b,\ b \neq 1)$ $\quad\quad$ $\int b^x\, dx = \frac{b^x}{\ln b} + C \quad (0 < b,\ b \neq 1)$

11. $\frac{d}{dx}[\ln|x|] = \frac{1}{x}$ $\quad\quad$ $\int \frac{1}{x}\, dx = \ln|x| + C$

12. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1 + x^2}$ $\quad\quad$ $\int \frac{1}{1 + x^2}\, dx = \tan^{-1} x + C$

13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{x^2 - 1}}$ $\quad\quad$ $\int \frac{1}{\sqrt{x^2 - 1}}\, dx = \sin^{-1} x + C$

14. $\frac{d}{dx}[\sec^{-1}|x|] = \frac{1}{x\sqrt{1 - x^2}}$ $\quad\quad$ $\int \frac{1}{x\sqrt{1 - x^2}}\, dx = \sec^{-1}|x| + C$

Theorem

$$\frac{d}{dx}\left[\int f(x)dx\right]=f(x)$$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int [f(x)\mp g(x)]dx = \int f(x)dx \mp \int g(x)dx$$

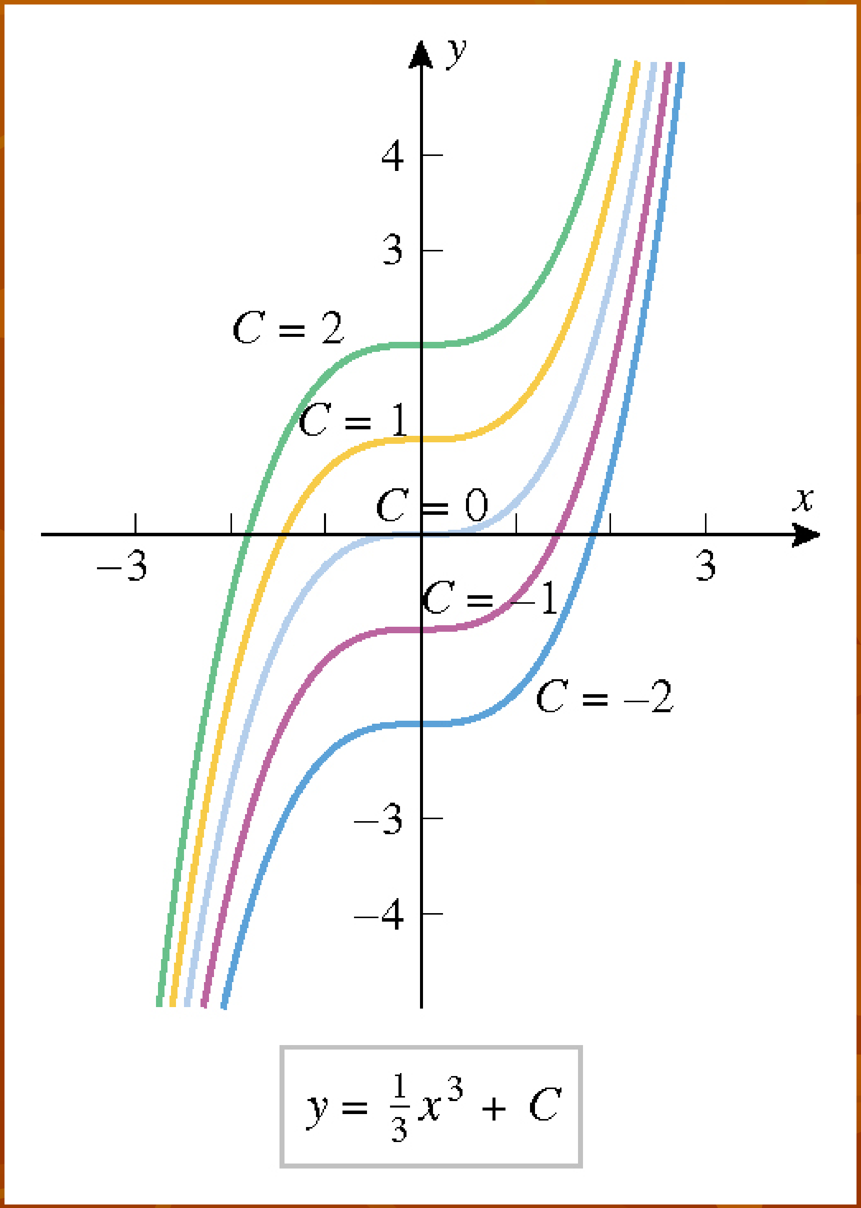
Example2 Evaluate

$a)\int \left(x+2x^2\right)dx$ $b)\int (x+2)^2 dx$ $c)\int \frac{x+\sqrt{x}}{x}dx$ $d)\int \frac{\sin x}{\cos^2 x}dx$

Integral Curves

If $y=F(x)+c$ is the antiderivative of $f(x)$ then the graph of $y=F(x)+c$ is called integral curves.

$$y = \int x^2 dx = \frac{x^3}{3} + c$$



Differential Equations

$$\frac{dy}{dx} = f(x),\ y(x_0) = y_0$$

Is called differential equation, initial-value problem

Example 3. Solve $\dfrac{dy}{dx} = \dfrac{x+1}{\sqrt{x}};\ y(0) = 1$