8.6 **Matches Requiring Special Substitutions**

(A) $u = x^{\frac{1}{n}}$ in which *n* is the LCD of the exponents.

Example 1 Evaluate

$$a)\int_0^1 \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx \qquad b)\int \frac{dx}{2+2\sqrt{x}} \qquad c)\int \sqrt{1+e^x}\,dx$$

(B) **Rational Function of *sin x and cos x***

Some examples are

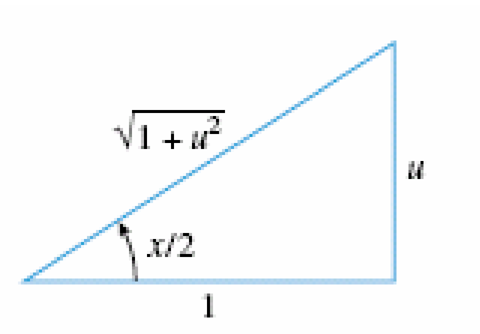
$$\frac{\sin x + 3\cos^2 x}{\cos x + 4\sin x}, \quad \frac{\sin x}{1+\cos x - \cos^2 x}, \quad \frac{3\sin^5 x}{1+4\sin x},$$

$$u = \tan\frac{x}{2}, \quad -\pi/2 < x/2 < \pi/2 \qquad x = 2\tan^{-1}u, \quad dx = \frac{2}{1+u^2}du$$

To express *sin x* and *cos x* in terms of *u*, we will use

$$\sin x = 2\sin(x/2)\cos(x/2)$$

$$\cos x = \cos^2(x/2) - \sin^2(x/2)$$



$$\sin(x/2) = \frac{u}{\sqrt{1+u^2}} \ \textit{and}\ \cos(x/2) = \frac{1}{\sqrt{1+u^2}}$$

$$\sin x = 2\left(\frac{u}{\sqrt{1+u^2}}\right)\left(\frac{1}{\sqrt{1+u^2}}\right) = \frac{2u}{1+u^2}$$

$$\cos x = \left(\frac{1}{\sqrt{1+u^2}}\right)^2 - \left(\frac{u}{\sqrt{1+u^2}}\right)^2 = \frac{1-u^2}{1+u^2}$$

$$\sin x = \frac{2u}{1+u^2}, \ \cos x = \frac{1-u^2}{1+u^2}, \ dx = \frac{2}{1+u^2}du$$

Example 1 Evaluate

$$\int \frac{1}{1-\sin x+\cos x}\, dx$$