[1] Find the Riemann Sum for $f(x) = \frac{15}{x}$, $1 \le x \le 3$ with 4 sub-intervals, taking sample points to be right end points

$\Rightarrow \Delta x = \frac{3-1}{4} = \frac{1}{2}$, $x_i = a + i\Delta x = 1+\frac{1}{2}, 1+1, 1+\frac{3}{2}, 1+2$

$RS = \sum_{i=1}^{4} f(x_i)\cdot\Delta x = \left[\frac{15}{\frac{3}{2}} + \frac{15}{2} + \frac{15}{\frac{5}{2}} + \frac{15}{3}\right]\left(\frac{1}{2}\right) = \frac{57}{4}$

[2] Show that the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \cdot \tan\left(\frac{\pi}{4n} i + \frac{\pi}{3}\right)$ can be interpreted as the area of $y = \tan\left(x + \frac{\pi}{3}\right)$, $0 \le x \le \pi/4$

$\Rightarrow \Delta x = \frac{\pi}{4n}$ in $[0, \pi/4]$, $x_i = \frac{i\pi}{4n}$

$f(x_i) = \tan\left(\frac{i\pi}{4n} + \frac{\pi}{3}\right) = \tan(x_i + \pi/3) \Rightarrow$

$f(x) = \tan(x + \pi/3)$ in $0 \le x \le \pi/4$

[3] Write the limit $\lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\right)$ as integral.

$\Rightarrow$ we can see, $f(x_i) = \left(1 + \frac{2i}{n}\right)^3 \Rightarrow x_i = 1 + \frac{2i}{n}$

$\Rightarrow a = 1$, $\Delta x = \frac{2}{n} \Rightarrow a = 1$, $b = 3$

$\Rightarrow f(x) = x^3$ and $\frac{1}{2}\lim_{n\to\infty}\left[\frac{2}{n}\sum\left(1+\frac{2i}{n}\right)^3\right]$

$\Rightarrow \frac{1}{2}\int_1^3 x^3\,dx$ in $[1, 3]$

[4] Integrate $\int \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}}\,dx$

Hint $\Rightarrow \int\left(1 - \frac{x}{\sqrt{1-x^2}}\right)dx = \int 1\,dx - \int\frac{x}{\sqrt{1-x^2}}\,dx$

Let $1 - x^2 = u \Rightarrow -2x\,dx = du$

ANS: $x + \sqrt{1-x^2} + C$

[5] Evaluate $\int_0^{\pi} |\sin 2x|\,dx$

$= \int_0^{\pi/2} \sin 2x\,dx - \int_{\pi/2}^{\pi} \sin 2x\,dx$

$\left\{ \begin{array}{l} |\sin 2x| = \sin 2x \\ 0 \le 2x \le \pi \\ \Rightarrow 0 \le x \le \pi/2 \\ \text{and } = -\sin 2x \\ \pi/2 < x \le \pi \end{array} \right.$

$= -\frac{1}{2}\left[\cos 2x\right]_0^{\pi/2} + \frac{1}{2}\left[\cos 2x\right]_{\pi/2}^{\pi}$

$= \boxed{2}$

(2)

[6] If $f(x) = \begin{cases} 3^x & 0 \le x \le 1 \\ 3^x + 1 & 1 < x \le 2 \end{cases}$, then $\int_0^2 f(x)\,dx$ is ______.

HINT $\Rightarrow \int_0^2 f(x)\,dx = \int_0^1 3^x\,dx + \int_1^2 (3^x + 1)\,dx$ = find it?

Remember $\int a^x\,dx = \frac{a^x}{\log a}$ ANSWER = $\left(\frac{8}{\log 3} + 1\right)$

* [7] If $x > e$ then $\frac{d}{dx}\left(\int_1^{\sqrt{\ln x}} e^{t^2}\,dt\right)$ is ...??

$$\Rightarrow \frac{d}{dx}\int^{\sqrt{\ln x}} e^{t^2}\,dt = \frac{d}{du}\int^{u} e^{t^2}\,dt \cdot \frac{du}{dx}, \quad u = \sqrt{\ln x}$$

$$\frac{du}{dx} = \frac{1}{2}\frac{1}{\sqrt{\ln x}} \cdot \frac{1}{x}$$

$$= e^{u^2} \cdot \frac{1}{2}\frac{1}{\sqrt{\ln x}} \cdot \frac{1}{x}$$

$$= e^{(\ln x)} \cdot \frac{1}{2}\frac{1}{\sqrt{\ln x}} \cdot \frac{1}{x} = \boxed{\frac{1}{2\sqrt{\ln x}}}$$

[8] Find $\int \frac{\cos\theta + \cos\theta \cdot \cot^2\theta}{\csc^2\theta}\,d\theta$

$$= \int (\sin^2\theta\cos\theta + \cos^3\theta)\,d\theta$$

$$= \int (\sin^2\theta + \cos^2\theta)\cos\theta\,d\theta = \int \cos\theta\,d\theta$$

$$= \boxed{\sin\theta + C}$$

[9] $\int \frac{5}{\sqrt{x}\,(3\sqrt{x}+4)^{3/5}}\,dx$ ANS $\frac{25}{3}(3\sqrt{x}+4)^{2/5} + C$

Hint Let $3\sqrt{x} + 4 = u$

[10] Find $F'(x)$ if $F(x) = \int_{2\ln x}^{x} \frac{e^t}{t}\,dt$

$$F'(x) = \frac{d}{dx}\left(\int_0^x \frac{e^t}{t}\,dt - \int_0^{2\ln x}\frac{e^t}{t}\,dt\right) =$$

$$= \frac{e^x}{x} - \frac{d}{dx}\left(\int_0^{2\ln x} e^t/t\,dt\right) = \frac{e^x}{x} - \frac{d}{du}\int_0^u e^t/t\,dt \cdot \frac{du}{dx}$$

where $u = 2\ln x$, $\frac{du}{dx} = \frac{2}{x}$

$$= e^x/x - e^u/u \cdot \frac{2}{x} = \frac{e^x}{x} - \frac{e^{2\ln x}}{2\ln x}\cdot\frac{2}{x}$$

$(e^{2\ln x} = x^2)$

= $e^x/$ ... $x/$ ...

$\boxed{11}$ $\displaystyle\int_0^{\ln 5} |e^x-4|\,dx$

$= -\displaystyle\int_0^{\ln 4}(e^x-4)\,dx + \int_{\ln 4}^{\ln 5} e^x-4\,dx$ $\quad \left\{ \begin{array}{l} |e^x-4| = e^x-4 \\ \text{if } e^x-4 \geq 0 \\ \Rightarrow x \geq \ln 4 \\ \ln 4 \leq x \leq \ln 5 \end{array} \right.$

$= \left[-e^x+4x\right]_0^{\ln 4} + \left[e^x-4x\right]_{\ln 4}^{\ln 5}$

$= [-4+4\ln 4+1-0] + [5-4\ln 5-4+4\ln 4]$

$= -2+8\ln 4-4\ln 5$

$\boxed{12}$. $\displaystyle\int \sin^3\theta \sqrt[3]{\cos\theta}\,d\theta$

$= \displaystyle\int \sin^2\theta \cdot \sin\theta \sqrt[3]{\cos\theta}\,d\theta$, Let $\cos\theta = u$
$\quad -\sin\theta\,d\theta = du$

$= \displaystyle\int (1-u^2)(-1)\sqrt[3]{u}\,du$

$= \displaystyle\int (-u^{1/3}+u^{7/3})\,du$ ... Now you can complete ...

$\boxed{13}$ Which one of the following integrals exists by Fundamental Theorem of Calculus?

(a) $\displaystyle\int_0^{\pi/4} \cot(x+\pi/2)\,dx$ (b) $\displaystyle\int_0^1 \ln x\,dx$ (c) $\displaystyle\int_{\pi/4}^{\pi} \sec x\,dx$ & (d) $\displaystyle\int_{-5}^{5} \frac{2}{x^7}\,dx$

$\Rightarrow$ Integral $\displaystyle\int_a^b f(x)\,dx$ exists if $f(x)$ is continuous in $[a,b]$

$\Rightarrow$ None of the functions is continuous in $[a,b]$ except (a)

ANS: (a)

$\boxed{14}$. If velocity of particle is $V(t) = \underline{3t-2}$ (m/sec). Find the distance travelled in $\frac{1}{2} \leq t \leq 2$

$d = \displaystyle\int_{1/2}^{2} |V(t)|\,dt = -\int_{1/2}^{1}(3t-3)\,dt + \int_1^2 (3t-3)\,dt$

$= \boxed{\frac{15}{8}}$

$|3t-3| = 3t-3$
if $3t-3 \geq 0 \Rightarrow t \geq 1$
$\Rightarrow 1 \leq t \leq 2$
and
$|3t-3| = -(3t-3)$
$t < 1$

15 $\displaystyle\int_0^{3\pi/4} |\cos x|\, dx$ ANS = $2-\frac{1}{\sqrt{2}}$

$$\Rightarrow \int_0^{\pi/2} \cos x\, dx - \int_{\pi/2}^{3\pi/4} \cos x\, dx$$

$\left\{ \begin{array}{l} |\cos x| = \cos x \\ 0 \le x \le \pi/2 \\ \text{and} \quad = -\cos x \\ \pi/2 \le x \le 3\pi/4 \end{array} \right.$

~~Now you~~ Now you can complete ...

16 If f is continuous function such that
$\int_1^x e^{-t} f(t)\, dt = 3 + x\sin x$ for all x. Find $f(x)$

$$\Rightarrow \frac{d}{dx}\int_1^x e^{-t} f(t)\, dt = \frac{d}{dx}\left(3 + x\sin x\right) = x\cos x + \sin x$$

$$\Rightarrow e^{-x} f(x) = x\cos x + \sin x \Rightarrow f(x) = \boxed{e^{x}\left(x\cos x + \sin x\right)}$$

17 Use area under the curves to evaluate $\int_{-2}^{2} |x|\, dx + \int_{-2}^{2} \sqrt{4-x^2}\, dx$

$$\Rightarrow 2\int_0^2 x\, dx + 2\int_0^2 \sqrt{4-x^2}\, dx$$

$$= 4 + 2\left(\frac{\pi\, 2^2}{4}\right) = \boxed{4+2\pi}$$

18 $\displaystyle\int_0^1 \frac{x^3+x^2+x+1}{x+1}\, dx$ ...... ANS = $\frac{4}{3}$

$$\Rightarrow \int_0^1 \frac{x^2(x+1)+1(x+1)}{x+1}\, dx = \int_0^1 (x^2+1)\, dx \ldots\ldots$$

19 If $F(x) = \displaystyle\int_{x^2}^{x^3} \sqrt{1+t^2}\, dt$, Find $F'(2)$ ANS: $18-10\sqrt{2}$

$$\Rightarrow F'(x) = \frac{d}{dx}\left(\int_{x^2}^{x^3}\sqrt{1+t^2}\, dt\right) = -\frac{d}{dx}\left(\int_0^{x^2}\sqrt{1+t^2}\, dt\right) + \frac{d}{dx}\left(\int_0^{x^3}\sqrt{1+t^2}\, dt\right)$$

$$= -\frac{d}{du}\left(\int_0^{u}\sqrt{1+t^2}\, dt \cdot \frac{du}{dx}\right) + \frac{d}{dw}\left(\int_0^{w}\sqrt{1+t^2}\, dt \cdot \frac{dw}{dx}\right)$$

$$= -2x\sqrt{1+x^4} + 3x^2\sqrt{1+x^6}$$

$\underline{F'(\sqrt{2}) = 18 - 10\sqrt{2}}$

$\left\{ \begin{array}{l} x^2 = u \Rightarrow 2x = \frac{du}{dx} \\ x^3 = w \Rightarrow 3x^2 = \frac{dw}{dx} \end{array} \right.$

[20] Interpret limit $\lim \sum_{i=1}^{n} \left[\frac{4i}{n^2} + \frac{6}{n} \right]$ as area.

$\Rightarrow \lim \sum \left(\frac{2}{n}\right)\left[3 + \frac{2i}{n}\right] = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{i=1}^{n} \left(3 + \frac{2i}{n}\right)$

$\Rightarrow a = 3, \; \Delta x = \frac{2}{n}$ in $[3,5]$ $x_i = a + i\,\Delta x = 3 + \frac{2i}{n}$

$\Rightarrow f(x_i) = x_i \Rightarrow f(x) = x$ on $[3,5]$

$\Rightarrow \boxed{\int_3^5 f(x)\, dx}$

[21] Find $\lim \sum_{i=1}^{n} \left(\frac{4i}{n^2} + \frac{3}{n}\right)$

$\Rightarrow \lim_{n\to\infty} \left[\frac{4}{n^2} \sum_{i=1}^{n} i + \frac{3}{n} \sum_{i=1}^{n} i\right] = \lim_{n\to\infty} \left[\frac{4}{n^2} \cdot \frac{n(n+1)}{2} + \frac{3}{n} \cdot n\right]$

$= \boxed{5}$

[22] $\int \frac{\cos(\pi/x^2)}{x^3} dx$, Let $\frac{\pi}{x^2} = u$ ...... ANS: $-\frac{1}{2\pi} \sin\left(\frac{\pi}{x^2}\right) + c$

[23] $\int_1^{e^4} \frac{\sqrt{\ln x}}{x} dx$, Let $\ln x = u$ ANS $= \frac{16}{3}$

[24] Express $\lim_{n\to+\infty} \sum \frac{\pi}{3n}\left(\frac{\pi}{6} + \frac{i\pi}{3n}\right)$ as definite integral

$\Delta x = \frac{\pi}{3n}$, $a = \frac{\pi}{6}$

$\Rightarrow x_i = \frac{\pi}{6} + \frac{i\pi}{3n}$, Interval $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

$f(x_i) = \cot(x_i) \Rightarrow \boxed{\int_{\pi/6}^{\pi/2} \cot x \, dx}$

[25] If f is continuous and $\int_3^5 f(x) = 8$, then find $\int_0^1 f(2x+3)\, dx$ ANS $= 4$

Let $2x+3 = u \Rightarrow 2\,dx = du$

$\int_3^5 f(u) \cdot \frac{du}{2} = \frac{8}{2} = \boxed{4}$

$\boxed{26}$ $\displaystyle\int \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$

$\Rightarrow \displaystyle\int \cos^2\theta \sin\theta\,d\theta + \int \sin^3\theta\,d\theta$

$\Rightarrow \displaystyle\int \sin\theta\,\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1}\,d\theta = \boxed{-\cos\theta + C}$

$\boxed{27}$ If $f(x) = \displaystyle\int_1^{x^2+3x} (t^3+1)^{20}\,dt$, find $f'(0)$

$\Rightarrow \quad f'(x) = \dfrac{d}{dx}\displaystyle\int_1^{x^2+3x} (t^3+1)^{20}\,dt$

$= \dfrac{d}{du}\displaystyle\int_1^{u} (t^3+1)^{20}\cdot\dfrac{du}{dx}$, $\quad u = x^2+3x$, $\dfrac{du}{dx} = 2x+3$

$= (u^3+1)^{20}\cdot(2x+3)$

$= [(x^2+3x)^3+1]^{20}(2x+3)$

$\boxed{f'(0) = 3}$

$\boxed{28}$ Find $\displaystyle\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{1/2x}$ $\quad\left\{ e = \displaystyle\lim_{x\to 0}(1+x)^{1/x}\right.$

$\Rightarrow \displaystyle\lim_{u\to 0}(1+u)^{\frac{1}{2\cdot 2u}}$ $\quad$ where $\dfrac{x}{2} = u$

$\Rightarrow \left[\displaystyle\lim_{u\to 0}(1+u)^{\frac{1}{u}}\right]^{\frac{1}{4}} = \boxed{e^{\frac{1}{4}}}$

$\boxed{29}$ If $A = \displaystyle\int_0^{\pi/2}\sqrt{1+\sin x}\,dx$, then $\dfrac{\pi}{2} \le A \le \dfrac{\pi}{\sqrt{2}}$

$\Rightarrow \quad 0 \le \sin x \le 1 \quad x \in [0, \frac{\pi}{2}]$

$\Rightarrow \quad 1 \le 1+\sin x \le 2$

$\Rightarrow \quad 1 \le \sqrt{1+\sin x} \le \sqrt{2}$

$\Rightarrow \displaystyle\int_0^{\pi/2} dx \le \int_0^{\pi/2}\sqrt{1+\sin x}\,dx \le \int_0^{\pi/2}\sqrt{2}\,dx$

$\Rightarrow \boxed{\dfrac{\pi}{2} \le A \le \dfrac{\pi}{\sqrt{2}}}$

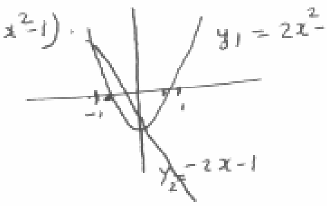
Sections 6.1, 6.2, 6.3

AREA and Volume

1 Find the area of the region enclosed by the curves $y = 2x^2 - 1$, $y = -2x - 1$ $\boxed{\text{ANS} = \frac{1}{3}}$

$\Rightarrow$ Point of intersection $2x^2 - 1 = -2x - 1 \Rightarrow x = 0, -1$

$$A = \int_{-1}^{0} (y_2 - y_1)\,dx = \int_{-1}^{0} (-2x-1) - (2x^2 - 1)$$

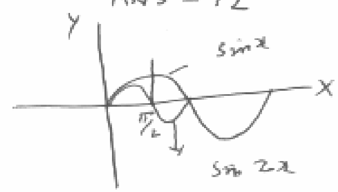


$$= \boxed{\frac{1}{3}}$$

2 Find the area enclosed by $y = \sin x$, $y = \sin 2x$, $x = 0$, $x = \frac{\pi}{2}$

ANS = $1/2$

$$A = \int_{0}^{\pi/2} (\sin x - \sin 2x)\,dx$$

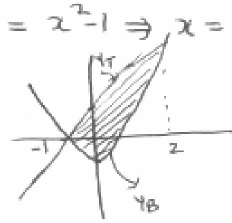


$$= \boxed{\frac{1}{2}}$$

3 Find the area between curves $y = x^2 - 1$, $y = x + 1$

$\Rightarrow$ Point of intersection $x + 1 = x^2 - 1 \Rightarrow x = -1, 2$

$$A = \int_{-1}^{2} \left[(x+1) - (x^2+1) \right] dx$$



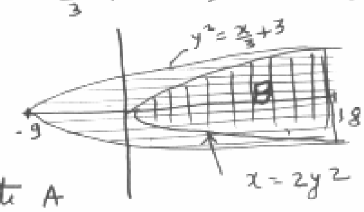
$$= \boxed{3}$$

4 Find the area of the region bounded by the graphs of $x = 2y^2$ and $y^2 = \frac{x}{3} + 3$

$\Rightarrow$ Point of Intersection $\frac{x}{2} = \frac{x}{3} + 3 \Rightarrow x = 18, y = 0$

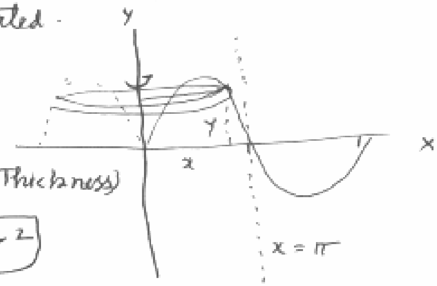
$$A = \int_{0}^{18} \sqrt{\frac{x}{2}}\,dx$$

$$B = \int_{-9}^{18} \sqrt{\frac{x}{3} + 3}$$



Area $= 2(B - A)$. Calculate A and B to get area.

[5] If the region enclosed by $y = \sin x$, $y = 0$, $x = 0$, $x = \pi$ is revolved about y-axis. Find the volume of solid generated.



$$V = \int_0^{\pi} 2\pi\, x\, y\, dx$$

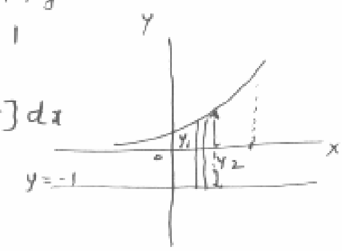
(2π * radius * height * Thickness)

$$= 2\pi \int_0^{\pi} x \sin x\, dx = \boxed{2\pi^2}$$

[6] The region $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about $y = -1$. Find the volume of solid formed. $\boxed{\text{ANS} = 7\pi/2}$

outer radius about $y = -1$ is $r_2 = 1 + y$

Inner . . . . $r_1 = 1$

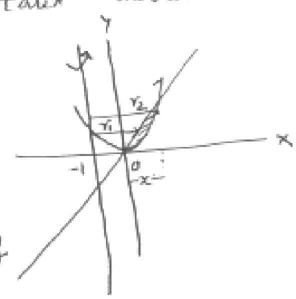


$$V = \int_0^{\ln 2} \pi \left[r_2^2 - r_1^2 \right] dx = \pi \int_0^{\ln 2} \left[(1+y)^2 - 1^2 \right] dx$$

$$= \pi \int_0^{\ln 2} \left(e^{2x} + 2e^x \right) dx$$

$$= \boxed{7\pi/2}$$

[7] If the region $y = x$, $y = x^2$ is rotated about $x = -1$, find the volume of the solid formed.



$$r_2 = x + 1 = \sqrt{y} + 1$$

$$r_1 = x + 1 = y + 1$$

$$V = \int_0^1 \pi \left(r_2^2 - r_1^2 \right) dy =$$

$$= \int_0^1 \pi \left((\sqrt{y}+1)^2 - (y+1)^2 \right) dy$$

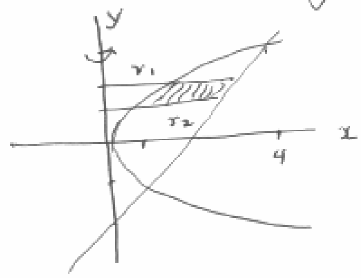
$$= \frac{\pi}{2}$$

[Point of intersection
$x = 0$, $x = 1$
$y = 0$, $y = 1$]

8 Write the integral for the volume of solid obtained by rotating the region bounded by $y^2 = x$, $y = x-2$ about y-axis

Point of intersection

$$(x-2)^2 = x$$
$$x^2 - 4x + 4 - x = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$\Rightarrow x = 1,\ x = 4$$
$$y = -1,\ y = 2$$



$r_2 =$ [illegible]

$$r_2 = x = y + 2$$
$$r_1 = x = y^2$$

$$V = \int_{-1}^{2} (r_2^2 - r_1^2)\, dy = \pi \int_{-1}^{2} ((y+2)^2 - y^4)\, dy$$

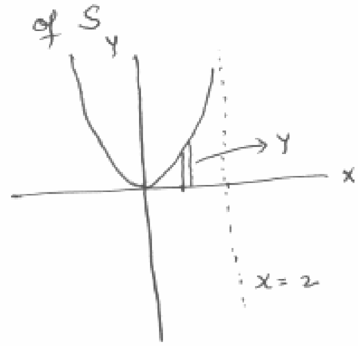
$$\boxed{V = \pi \int_{-1}^{2} (y^2 + 2y + 4 - y^2)\, dy}$$

9 Write the integral for the volume generated by rotating the region bounded by $y = \ln x$, $y = 0$, $x = e$ about y- axis

$$r_2 = e,\ r_1 = x$$

$$V = \int_0^1 \pi (r_2^2 - r_1^2)\, dy$$
$$= \pi \int_0^1 (e^2 - x^2)\, dy = \boxed{\pi \int_0^1 (e^2 - e^{2y})\, dy}$$

10 The base of a solid is enclosed by the curves $y = x^2$, $y = 0$, $x = 2$. If Cross-sections of S are perpendicular to the x- axis are squares. Find the volume of S



Area of squares $= y^2$

Volume of the square $= y^2\, dx$

$$V = \int_0^2 y^2\, dx = \int_0^2 x^4\, dx$$
$$= \boxed{\frac{32}{5}}$$