

TUTORIAL Chapter 15

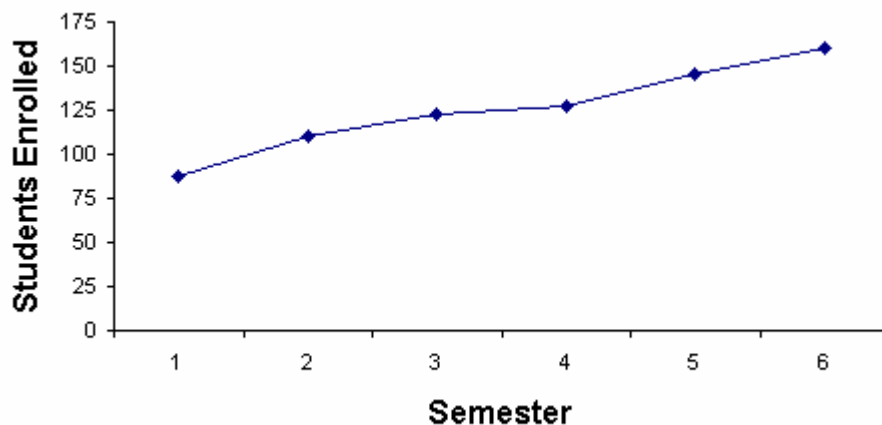
EXAPMLE 1.(34)

The following data represents enrollment in a major in one university for the past 6 semesters.

Semester	1	2	3	4	5	6
Enrollment	87	110	123	127	145	160

- Construct a graph of the time series of the enrollment.
- Does it appear a trend in the enrollment data?
- Prepare a simple exponential smoothing forecast for semester 7 using $\alpha=0.35$ and $Y_1 = 87$.
- Prepare a double exponential smoothing forecast using $\alpha=0.20$ and $\beta=0.25$.
- Calculate MAD for both forecasts. Which model appears to be doing better?

a)



- Yes it does appear that trend is present.

c. % Which Equation do we use?

$$F_{t+1} = F_t + \alpha(y_t - F_t)$$

Semester	Actual Enrollment	Forecast Enrollment	Forecast Error	Absolute Forecast Error
1	87	87.00	0.00	0.00
2	110	87.00	23.00	23.00
3	123	95.05	27.95	27.95
4	127	104.83	22.17	22.17
5	145	112.59	32.41	32.41
6	160	123.93	32.07	32.07
7		136.56		
			Sum	139.67

Alpha	0.35
MAD	23.599

d. Which Equations do we use?

$$C_t = \alpha y_t + (1 - \alpha)(C_{t-1} + T_{t-1})$$

$$T_t = \beta(C_t - C_{t-1}) + (1 - \beta)T_{t-1}$$

$$F_{t+1} = C_t + T_t$$

Semester	Actual Enrollment	Constant	Trend	Forecast Enrollment	Forecast Error	Absolute Forecast Error
<i>Initial Values</i>		77.93	13.54			
1	87	90.58	13.32	91.48	-4.48	4.48
2	110	105.12	13.62	103.90	6.10	6.10
3	123	119.60	13.84	118.74	4.26	4.26
4	127	132.15	13.52	133.43	-6.43	6.43
5	145	145.53	13.48	145.66	0.66	0.66
6	160	159.21	13.53	159.01	0.99	0.99
7				172.74		
					Sum	22.91

Alpha	0.2	Initial Constant	13.54
Beta	0.25	77.93	10
MAD	3.819		

- e. The MAD for the single exponential smoothing forecast was 23.599
The MAD for the double exponential smoothing forecast was 3.819

The double exponential smoothing forecast appears to be doing the better job of forecasting course enrollment.

EXAMPLE 2. (25). The following sales are given in millions dollars.

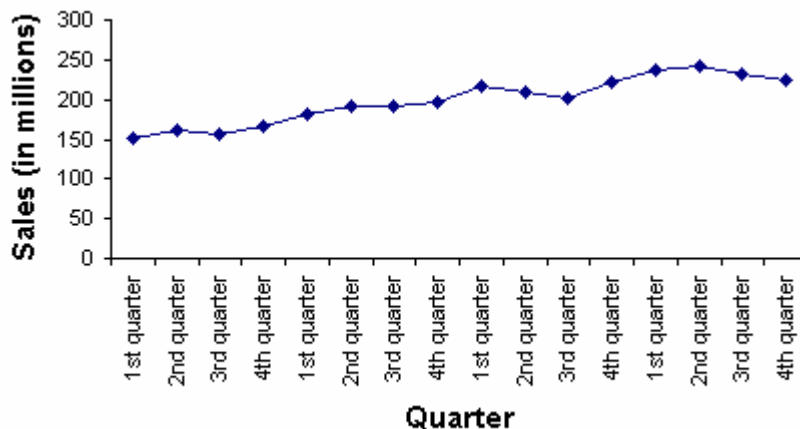
1997	Sales	1999	Sales
1	152	1	217
2	162	2	209
3	157	3	202
4	167	4	221

1998		2000	
1	182	1	236
2	192	2	242
3	191	3	231
4	197	4	224

- Plot these data. What time series components are presented in these data?
- Determine seasonal index for each quarter.
- Fit a linear trend model for deseasonalized data for 1997 through 2000 and determine MAD and MSE values. Comment on the adequacy of the linear trend model based on these measures of forecast error.
- Provide a seasonally adjusted forecasts using the linear trend model for each quarter of 2001.

Solution:

a.



There appears to be an upward linear trend but you also see a seasonal component as a slight drop in the 3rd quarter.

b.

Year	Quarter	Period	Actual Sales	4 Period Moving Average	Centered Moving Average	Ratio to MA	Deasonalized Sales
1997	1st quarter	1	152				146.8580
	2nd quarter	2	162	159.5			158.6838
	3rd quarter	3	157	167	163.25	0.961715	163.5528
	4th quarter	4	167	174.5	170.75	0.978038	169.6889
1998	1st quarter	5	182	183	178.75	1.018182	175.8431
	2nd quarter	6	192	190.5	186.75	1.028112	188.0697
	3rd quarter	7	191	199.25	194.875	0.980115	198.9719
	4th quarter	8	197	203.5	201.375	0.978274	200.1719
1999	1st quarter	9	217	206.25	204.875	1.059182	209.6591
	2nd quarter	10	209	212.25	209.25	0.998805	204.7217
	3rd quarter	11	202	217	214.625	0.941176	210.4311
	4th quarter	12	221	225.25	221.125	0.999435	224.5583
2000	1st quarter	13	236	232.5	228.875	1.031131	228.0164
	2nd quarter	14	242	233.25	232.875	1.039184	237.0462
	3rd quarter	15	231				240.6415
	4th quarter	16	224				227.6066

1) $(152+162+157+167)/4=159.5$

2) $(159.5+167)/2=163.25$

3) $157/163.25=0.961715$

4) $152/1.035013=146.8580$ Why? Where did we obtain 1.035013?

SEASONA INDEX: $(1.018182+1.059182+1.031131)/3=1.035013$

Because the seasonal index numbers do not add to 4, we normalize them by multiplying each by $4/4.00445$ to get the following values:

Quarter	Seasonal Index
1	1.035013
2	1.020898
3	0.959934
4	0.984154

c.

SUMMARY OUTPUT

<i>Regression Statistics</i>	
Multiple R	0.976793469
R Square	0.954125481
Adjusted R Square	0.950848729
Standard Error	6.498762579
Observations	16

ANOVA

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1	12297.68445	12297.684	291.1803	9.14173E-11
Residual	14	591.2748109	42.233915		
Total	15	12888.95927			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	147.91253	3.407979848	43.401821	2.5E-16
Quarter	6.014121728	0.352444885	17.064006	9.14E-11

RESIDUAL OUTPUT

<i>Observation</i>	<i>Predicted Deseasonalized Sales</i>	<i>Residuals</i>	<i>Squared Residuals</i>	<i>Absolute Value</i>
1	153.9266518	-7.068650089	49.965814	7.06865
2	159.9407735	-1.256975545	1.5799875	1.256976
3	165.9548952	-2.402046206	5.769826	2.402046
4	171.9690169	-2.280136662	5.1990232	2.280137
5	177.9831387	-2.140005093	4.5796218	2.140005
6	183.9972604	4.07242605	16.584654	4.072426
7	190.0113821	8.960555203	80.29155	8.960555
8	196.0255039	4.146408811	17.192706	4.146409
9	202.0396256	7.619495222	58.056707	7.619495
10	208.0537473	-3.332057374	11.102606	3.332057
11	214.067869	-3.6368149	13.226423	3.636815
12	220.0819908	4.476347808	20.03769	4.476348
13	226.0961125	1.920258519	3.6873928	1.920259
14	232.1102342	4.935933072	24.363435	4.935933
15	238.124356	2.51709705	6.3357776	2.517097
16	244.1384777	-16.53183587	273.3016	16.53184
			MSE	MAD
			36.954676	4.831065

Values of the MSE and MAD are best used to compare two or more forecasting models. For this model the MAD, for instance indicates the average forecasting error

is less than five (million). For the final period of data this is about 2%, which might be considered acceptable.

e. %How we find forecast for period 17?

USE equation: $\text{sales} = 147.91253 + 6.014121728T$

%Why?

Example: $147.91253 + 6.014121728 \times 17 = 250.1526$.

Take your calculator and check all values in third column.

%How we find last column? CHECK!!!!

Third column \times seasonal index = last column

Quarter	Period	Seasonally Unadjusted Forecast	Seasonal Index	Seasonally Adjusted Forecast
Quarter 1 2001	17	250.1526	1.0350134	258.9113
Quarter 2 2001	18	256.1667	1.0208982	261.5201
Quarter 3 2001	19	262.1808	0.9599344	251.6764
Quarter 4 2001	20	268.1950	0.9841541	263.9452