Age-dependent branching processes with incubation

I. RAHIMOV

Department of Mathematical Sciences, KFUPM,
Box. 1339, Dhahran, 31261, Saudi Arabia
e-mail: rahimov @kfupm.edu.sa

Abstract

We consider a model of age-dependent branching stochastic process which takes into account incubation period of the life of individuals. We demonstrate that such processes may be treated as a two-type branching process with a periodic mean matrix. Based on this one can obtain extinction probability and asymptotic behavior of the mean number of individuals, when a Malthusian parameter exists. However in subcritical case the Malthusian parameter may not exist. In this case the study of the process needs more delicate analysis and requires additional restrictions on the life time distributions. We obtain certain properties of sub-exponential distributions, introduced by Chistyakov (1964), and describe a subclass of such distributions which is closed with respect to convolution. Using these results we derive asymptotic behavior of the first and second moments and of the probability of non-extinction. We also prove a limit theorem for the process conditioned on the non-extinction. Exact formulas for expected extinction time and for distribution of the number of generations to extinction will be obtained. Applications in determining of the optimal vaccination rate in SIR epidemics will also be discussed.