King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics

MATH - 202   Semester 062   Major Examination - II

Venue: Bldg. 5, Room 201
Time: 6:30 p.m. to 8:00 p.m.
Date: April 25, 2007
Max. Marks: 40

Name:_____

ID#:_____

S. No#:_____

Section#:_____

Instructions:

1. *Programmable calculators and mobile phones are* **NOT** *allowed in the examination hall.*

2. *Clearly indicate the theorem/result while applying it to solve a problem.*

3. *Write all calculations in the answer sheet.*

Instructor: Dr. Qamrul Hasan Ansari

1. Verify that e^{-3x}, e^{4x} form a fundamental set of solutions of $y'' - y' - 12y = 0$ on $(-\infty, \infty)$.

(4 marks)

Solution:

2. Let $y_{p_1} = 3e^{2x}$ and $y_{p_2} = x^2 + 3x$ be particular solutions of ODEs $y'' - 6y' + 5y = -9e^{2x}$ and $y'' - 6y' + 5y = 5x^2 + 3x - 16$, respectively. Use these solutions to find particular solution of $y'' - 6y' + 5y = 5x^2 + 3x - 16 - 9e^{2x}$. (5 marks)

   <u>Solution:</u>

3. One solution of ODE $y'' - 3y' + 2y = 5e^{3x}$ is $y_1 = e^x$. Find the second solution and general solution of ODE by reduction of order method.   (4 marks)

   Solution:

4. Solve $\dfrac{d^4y}{dx^4} + 2\dfrac{d^2y}{dx^2} + y = 0$.   (5 marks)

   Solution:

5. Determine the form of a particular solution for $y''' - 4y'' + 4y' = 5x^2 - 6x + 4x^2 e^{2x} + 3e^{5x}$.

(4 marks)

Solution:

6. If $y_{1_c} = e^{-x}$ and $y_{2_c} = e^{-2x}$ are complementary solutions of ODE $y'' + 3y' + 2y = \dfrac{1}{(1 + e^x)}$, then find the general solution of ODE.   (5 marks)

<div align="center">OR</div>

Solve $y'' - y = \dfrac{1}{x}$ by using variation of parameter.   (5 marks)

<u>Solution:</u>

7. Solve $x^3 y''' + xy' - y = 0$.   (5 marks)

<div align="center">OR</div>

Solve $x^2 y'' + 10xy' + 8y = x^2$ by reducing it to constant coefficient equation.   (5 marks)

Solution:

7. Solve $x^3 y''' + xy' - y = 0$.   (5 marks)

<div align="center">OR</div>

Solve $x^2 y'' + 10xy' + 8y = x^2$ by reducing it to constant coefficient equation.   (5 marks)

Solution:

8. Verify that $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$ is a particular solution of ODE $xy'' + y' + xy = 0$.   (4 marks)

Solution:

9. Formulate the recursive relation to find the constants in the general solution of ODE $y'' - 2xy' + y = 0$ about the ordinary point $x = 0$. (Do not find the solution of given ODE). (4 marks)

OR

If $c_{k+2} = -\dfrac{c_{k-1}}{(k+1)(k+2)}$ for $k = 1, 2, ...$ is the recursive relation, then find the power series solution of ODE $y'' + xy' = 0$. (4 marks)

Solution:

For Rough Work

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