

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematical Sciences**

Math - 535 Semester - 061 Major Exam # I

**Name:**

**S. No.:**

**ID:**

Maximum Marks: 40

Section:

Time Allowed: 90 Minutes

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**NOTE:** Give the solution of any SIX questions.

1. Show that the real line  $\mathbb{R}$  is not compact. (7 Marks)
2. Prove that a subspace  $Y$  of a complete normed space  $X$  is complete if and only if  $Y$  is closed in  $X$ . (6 Marks)
3. If  $X$  is a finite dimensional normed space, then prove that every linear operator defined on  $X$  is bounded. (6 Marks)
4. Show that the continuity and boundedness are equivalent for a linear operator  $T$  defined on a normed space  $X$  into another normed space  $Y$ . (7 Marks)
5. State uniform boundedness principle and show that it is not valid if the space  $X$  is only a normed space. (6 Marks)
6. If  $T(x) = T(y)$  for every bounded linear functional  $T$  defined on a normed space  $X$ , then prove that  $x = y$ . (6 Marks)
7. Give the definition of a closed operator  $T$  defined on a normed space  $X$  into another normed space  $Y$ . Also, give the characterization of closed operator. (7 Marks)
8. Is every closed operator continuous and vice-versa? Justify your answer. (7 Marks)

**Solution.**