King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math - 201 Semester - 061 Quiz # I

Name: **S. No.:** **ID:**

Maximum Marks: 10 **Solution** Time Allowed: 20 minutes

NOTE: *Give the solution of any TWO questions.*

1. Sketch the curve $r = 4(1 - \cos\theta)$ and set up the integral to evaluate the arc length of this curve. (Simplify the integral but do not integrate it) (5 Marks)
2. Sketch the polar curve $r = 4\cos 3\theta$ and set up the integral to find the area enclosed by it. (Simplify the integral but do not integrate it) (5 Marks)
3. Sketch the polar curve $r = 1 + 2\cos\theta$ and find the polar equations of the tangent lines to this curve at pole. (5 Marks)

Solution 1. since $r = 4(1 - \cos\theta)$, $\frac{dr}{d\theta} = 4\sin\theta$ and $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 32(1 - \cos\theta)$. The arc length $= \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = 2\int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = 2\int_0^{\pi} \sqrt{32(1 - \cos\theta)} d\theta.$

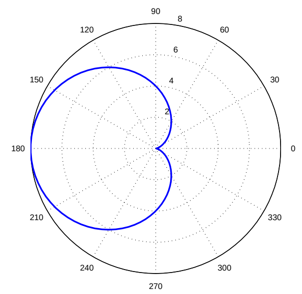

Figure 1: $r = 4(1 - \cos\theta)$

Solution 2. When $r = 0$, $\theta = \pi/6,\ \pi/2$. Therefore,

$$\text{area } = 6\int_0^{\pi/6} \frac{1}{2} r^2 d\theta = 48\int_0^{\pi/6} \cos^2 3\theta d\theta.$$

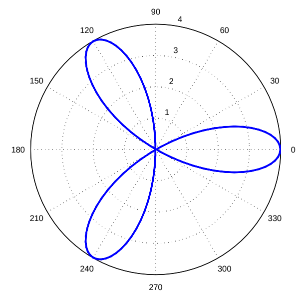

Figure 2: $r = 4\cos 3\theta$

Solution 3. When $r = 0$, the curve passes through the origin. Therefore, $1 + 2\cos\theta = 0 \Rightarrow \theta = 2\pi/3,\ 4\pi/3$. The curve passes through the pole at $\theta = 2\pi/3,\ 4\pi/3$.

$\frac{dr}{d\theta} = -2\cos\theta$ and $\frac{dr}{d\theta}|_{\theta=2\pi/3,\ 4\pi/3} \neq 0$. Thus, $\theta = 2\pi/3$ and $\theta = 4\pi/3$ are tangent lines to the curve at pole.

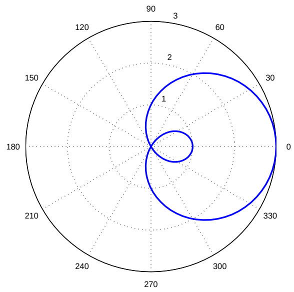

Figure 3: $r = 1 + 2\cos\theta$