4.2 Reduction of Order

Objectives:

- To learn a method of solving 2nd order ODEs by reducing their order.
- This method is based on using one known solution to reduce the order and then find the second solution.

- Consider a second order homogeneous linear ODE

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0 \qquad (1)$$

- The general solution of ODE (1) is $y = c_1 y_1 + c_2 y_2$, where y_1 and y_2 are solutions that constitute a linearly independent set on some interval.
- Divide (1) by $a_2(x)$ in order to put equation (1) in the **standard form**

$$y'' + P(x)y' + Q(x)y = 0 \qquad (2)$$

where $P(x)$ and $Q(x)$ are continuous on some interval.

- Assume that $y_1(x)$ is a known solution of (2) and that $y_1(x) \neq 0$ for every x in that interval.
- Second solution can be found by the following formula

$$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)} dx \qquad (3)$$

- The general solution of (1) is $y = c_1 y_1 + c_2 y_2$.

We shall learn the process to find formula (3) by examples.

Question 9/142: One solution of $x^2y'' - 7xy' + 16y = 0$ is $y_1(x) = x^4$. Find the second solution and general solution of this ODE by reduction of order method.

Question 13/142: One solution of $x^2y'' - xy' + 2y = 0$ is $y_1(x) = x\sin(\ln x)$. Find the second solution and general solution of this ODE by reduction of order method.

Question 17/142: One solution of $y'' - 4y = 12$ is $y_1(x) = e^{2x}$. Find the second solution and general solution of this ODE by reduction of order method.