10.1 Curves Defined by Parametric Equations

**Parametric Equations & Parametric Curves**

- **Parametric Equations:** If $x$ and $y$ are both functions of a third variable $t$, that is,
  \[ x = f(t) \quad y = g(t), \]

  Then these equations are called *parametric equations* and the third variable $t$ is called *parameter*.
- Each value of $t$ determines a point $(x, y)$, which we can plot in a coordinate plane.
- As $t$ varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve, called *parametric curve*.
- The curve with parametric equations
  \[ x = f(t) \quad y = g(t), \quad a \leq t \leq b \]
  has initial point $(f(a), g(a))$ and the terminal point $(f(b), g(b))$.
- A curve may have more than one set of parametric equations, for example,
  \[ x = \cos t, \quad y = \sin t \quad \text{and} \quad x = \sin 2t, \quad y = \cos 2t \]
  represent circle with center at origin and radius one.
- The graph of the parametric curve can be determined by plotting $(x, y)$ corresponding to parameter $t$.
- We can always convert the parametric equations in the Cartesian equation.
- We can convert the Cartesian equation $y = f(x)$ in the parametric equations by taking $x = t$ and $y = f(t)$. 
Question 10/656: Sketch the curve by using the parametric equations
\[ x = t^2, \quad y = t^3 \] to plot points. Also, estimate the parameter to find a Cartesian equation of the curve.

Question 13/656: Estimate the parameter to find a Cartesian equation of the curve \[ x = \sin^2 \theta, \quad y = \cos^2 \theta. \] Also, sketch the curve.

Question 36/658: Find three different sets of parametric equations to represent the curve \[ y = x^3, \quad x \in \mathbb{R}. \]