10.1 Curves Defined by Parametric Equations

Parametric Equations & Parametric Curves

- **Parametric Equations:** If x and y are both functions of a third variable t, that is,

$$x = f(t) \qquad y = g(t),$$

Then these equations are called ***parametric equations*** and the third variable t is called ***parameter***.

- Each value of t determines a point (x, y), which we can plot in a coordinate plane.
- As t varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve, called ***parametric curve***.
- The curve with parametric equations

$$x = f(t) \qquad y = g(t), \qquad a \le t \le b$$

has initial point $(f(a), g(a))$ and the terminal point $(f(b), g(b))$.

- A curve may have more than one set of parametric equations, for example,

$$x = \cos t, \quad y = \sin t \quad \text{and} \quad x = \sin 2t, \quad y = \cos 2t$$

represent circle with center at origin and radius one.

- The graph of the parametric curve can be determined by plotting (x, y) corresponding to parameter t.
- We can always convert the parametric equations in the Cartesian equation.
- We can convert the Cartesian equation $y = f(x)$ in the parametric equations by taking $x = t$ and $y = f(t)$.

Question 10/656: Sketch the curve by using the parametric equations $x = t^2$, $y = t^3$ to plot points. Also, estimate the parameter to find a Cartesian equation of the curve.

Question 13/656: Estimate the parameter to find a Cartesian equation of the curve $x = \sin^2\theta$, $y = \cos^2\theta$. Also, sketch the curve.

Question 36/658: Find three different sets of parametric equations to represent the curve $y = x^3$, $x \in \mathbb{R}$.