15.7 Triple Integrals

Triple integrals

Notation: $\iiint\limits_G f(x,y,z)\,dV$

Where G is the solid and $dV = dxdydz$ or any other seven forms.

All rules are same as in case of double integral.

Dfinition

Let $G = \{(x,y,z) : a \le x \le b, c \le y \le d, r \le z \le s\}$ be the rectangular box. Divide this box into lmn sub-boxes $G_{ijk} = [x_{i-1}, x_i] \times [y_{j-1}, y_j] \times [z_{k-1}, z_k]$. Then the triple integral over The box G is

$$\iiint\limits_G f(x,y,z)\,dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x^*_{ijk}, y^*_{ijk}, z^*_{ijk})\Delta x\,\Delta y\,\Delta z$$

if this limit exists

Evaluating Triple Integrals over Rectangular Boxes

Evaluated as iterated integrals

Let G be the rectangular box $a \le x \le b,\ c \le y \le d,\ r \le z \le s$ then

$$\iiint_G f(x,y,z)\,dV = \int_a^b \int_c^d \int_r^s f(x,y,z)\,dz\,dy\,dx$$

- Or any other ordering with proper adjustment of limits of integration.

Question 6/1030: Evaluate $\int_0^1 \int_0^z \int_0^y z e^{-y^2}\,dx\,dy\,dz$.

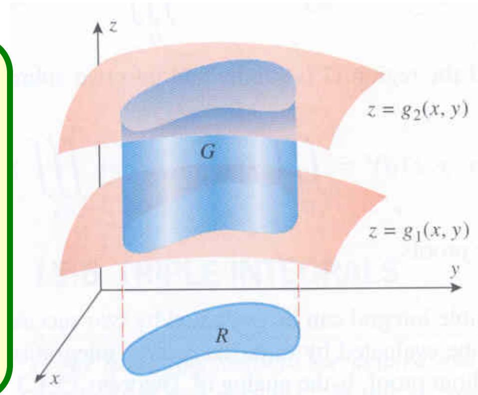
Evaluating Triple Integrals over Simple XY-region

Simple XY-region

A region G given by

- $G = \{(x,y,z) : (x,y) \in R \text{ and } g_1(x,y) \le z \le g_2(x,y)\}$

where R is projection of G on XY-plane



Evaluation of triple integral over such regions as

- $$\iiint_G f(x,y,z)\,dV = \iint_R \left[\int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)\,dz \right] dA$$

Evaluating Triple Integrals over Simple YZ-region

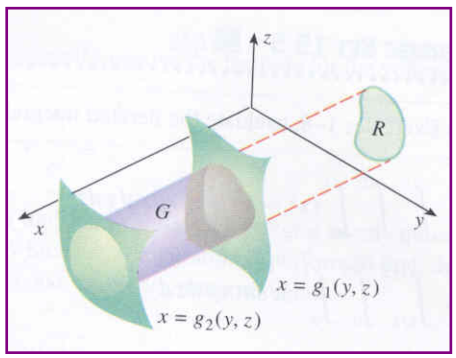
See class explanation

Simple YZ-region

A region G given by

- $G = \{(x,y,z) : (y,z) \in R \text{ and } g_1(y,z) \le x \le g_2(y,z)\}$

where R is projection of G on YZ-plane



Evaluation of triple integral over such regions as

- $$\iiint_G f(x,y,z)\,dV = \iint_R \left[\int_{g_1(y,z)}^{g_2(y,z)} f(x,y,z)\,dx \right] dA$$

Evaluating Triple Integrals over Simple XZ-region

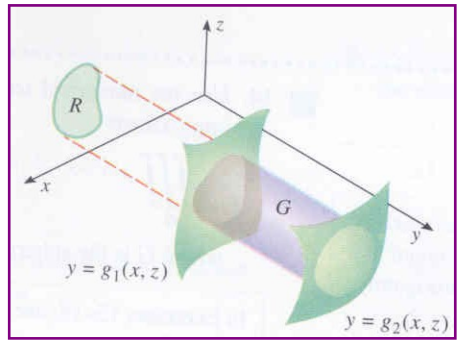
See class explanation

Simple XZ-region

A region G given by

- $G = \{(x,y,z) : (x,z) \in R \text{ and } g_1(x,z) \le y \le g_2(x,z)\}$

where R is projection of G on XZ-plane



Evaluation of triple integral over such regions as

- $$\iiint_G f(x,y,z)\,dV = \iint_R \left[\int_{g_1(x,z)}^{g_2(x,z)} f(x,y,z)\,dy\right] dA$$

Question 8/1030: Evaluate $\iiint_G 2x\,dV$

where $G = \{(x,y,z) : 0 \le x \le 1, 0 \le y \le x, x \le z \le 2x\}$.

Question 13/1030: Evaluate $\iiint_G x^2 e^y dV$, where G is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $z = 0$, $x = 1$, and $x = -1$.

- Try to project the region *G* on XY, YZ, XZ planes
- and see which gives you easier calculations and better visualization of *G* & *R*.
- **Easiest:** thinking as integral over XY-region

Question 16/1030: Evaluate $\iiint_G z dV$, where G is bounded by the cylinder $z^2 + y^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$ in the first octant.

Volume as Triple Integral

The volume of a 3-dimensional region G is given by

$$V = \iiint_G dV$$

Question 17/1054: Use triple integral to find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$.

Question 34/1055: Reverse the order in the form $dxdzdy$.

$$\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z)\, dz\, dy\, dx .$$