15.3 Double Integrals over General Region

Evaluating Double Integrals over General (Non-Rectangular) Regions

We will consider two types of regions

Type I region

A region bounded by

- the lines $x = a, x = b$

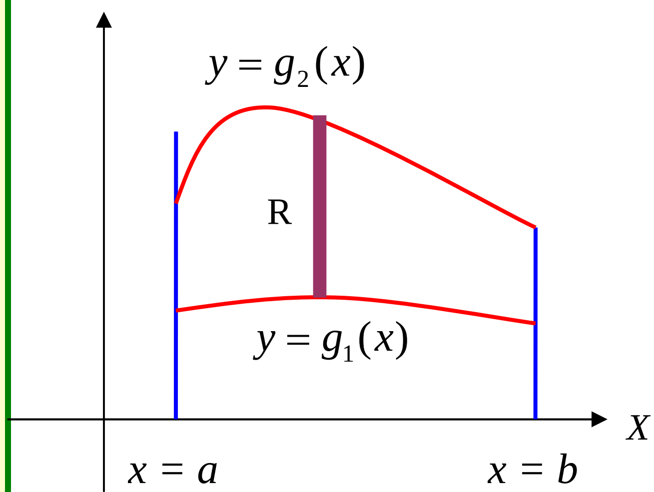
and

- the curves $y = g_1(x), \; y = g_2(x)$

  with $g_1(x) \le g_2(x) \quad \forall x \in [a,b]$

Type I region is

$$R_V = \{(x,y): a \le x \le b, \; g_1(x) \le y \le g_2(x)\}$$



Type II region

A region bounded by

- the lines $y = c, y = d$

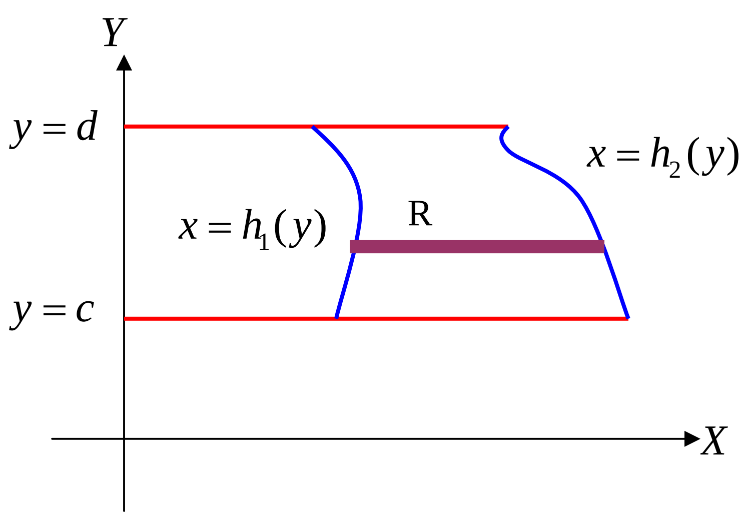
and

- the curves $x = h_1(y), \; x = h_2(y)$

  with $h_1(y) \le h_2(y) \quad \forall y \in [c,d]$

Type II region is

$$R_H = \{(x,y): c \le y \le d, h_1(y) \le x \le h_2(y)\}$$



Double integrals over both regions evaluated as iterated integrals

If R is type I then

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx$$

If R is type II then

$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dxdy$$

Find limit of Integrals for type I region R

Step 1. Draw a vertical line through the region R at an arbitrary fixed value *x*. This line intersects the region below at the curve $y = g_1(x)$ and above at the curve $y = g_2(x)$. Then $y = g_1(x)$ is the lower limit and $y = g_1(x)$ is the upper limit of the inner integral.

Step 2. Move the line left and then right. Leftmost position where the line intersects the region R is $x = a$ which is lower limit of the outer integral. Rightmost position where the line intersects the region R is $x = b$ which is the upper limit of the outer integral.

Find limit of Integrats for type II region R

Step 1. Draw a horizontal line through the region R at an arbitrary fixed value *y*. This line intersects the region left at the curve $y = h_1(x)$ and right at the curve $y = h_2(x)$. Then $y = h_1(x)$ is the lower limit and $y = h_1(x)$ is the upper limit of the inner integral.

Step 2. Move the line down and then up. Lowest position where the line intersects the region R is $y = c$ which is lower limit of the outer integral. Highest position where the line intersects the region R is $y = d$ which is the upper limit of the outer integral.

Important points to note:

- Outer limits always constant
- The idea of using a vertical line as explained in class
- Sometimes need to break into more integrals.

Question 5/1002: Evaluate $\int_0^{\pi/2}\int_0^{\cos\theta} e^{\sin\theta} dr d\theta$.

Question 9/1002: Evaluate the double integral $\iint_R \frac{2y}{x^2+1} dA$ where $R = \{(x, y) : 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$.

Question 17/1002: Evaluate the double integral $\iint_R (2x - y) dA$; R is bounded by the circle with center the origin and radius 2.

Question 22/1002: Use double integral to find the volume of the solidenclosed by the paraboloid $z = x^2 + 3y^2$ and the plane $x = 0, y = 1, y = x, z = 0$.

Question 28/1002: Find the volume of the region bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.

Question 42/1003: Sketch the region of integration and change the order of the integration $\int_0^1 \int_{\arctan\theta}^{\pi/4} f(x, y)dydx$.

Question 48/1003: Evaluate the integral $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dxdy$ by reversing the order of integration.