15.2 Iterated Integrals

Evaluating Double Integrals over Rectangular Regions

more general regions in next section

Partial Integration

$\int_a^b f(x,y)\,dx$ = partial definite integral w.r.t. x.

(It is evaluated by holding y fixed and integrating w.r.t x)

$\int_c^d f(x,y)\,dy$ = partial definite integral w.r.t. y.

(It is evaluated by holding x fixed and integrating w.r.t y)

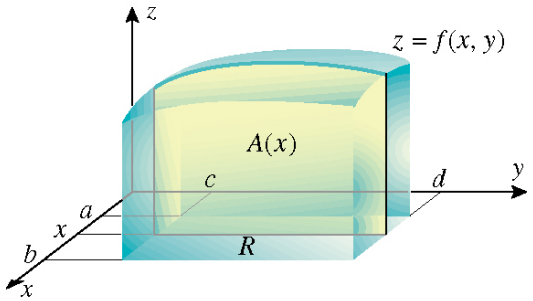
If the region R is defined by $a \le x \le b$, $c \le y \le d$ then

$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

i.e. R is rectangular region $[a,b]\times[c,d]$

Note:

- inner limits for inner differential
- outer limits for outer differential



These integrals are evaluated as iterated integrals

- $\int_c^d \int_a^b f(x,y)\,dx\,dy = \int_c^d \left[\int_a^b f(x,y)\,dx \right] dy$
- $\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_a^b \left[\int_c^d f(x,y)\,dy \right] dx$

Question 9/994: Calculate the iterated integral $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x} \right) dy dx$.

Question 14/994: Calculate the double integral $\iint_R \cos(x + 2y) dA$ over the region $R = \{(x, y) : 0 \le x \le \pi, \ 0 \le y \le \pi / 2\}$.

Question 27/994: Find the volume of the solid enclosed by the surface $z = x\sqrt{x^2 + y}$ and the planes $x = 3, x = 1, y = 0, y = 1$, and $z = 0$.

Question 34/1020: Find the average value of $f(x, y) = e^y \sqrt{x + e^y}$ over the rectangle $R = [0,4] \times [0,1]$ by using the following formula

$f_{ave} = \frac{1}{A(R)} \iint_R f(x, y) dA$ where $A(R)$ is the area of R .