15.1 Double integrals over Rectangles

Motivation:

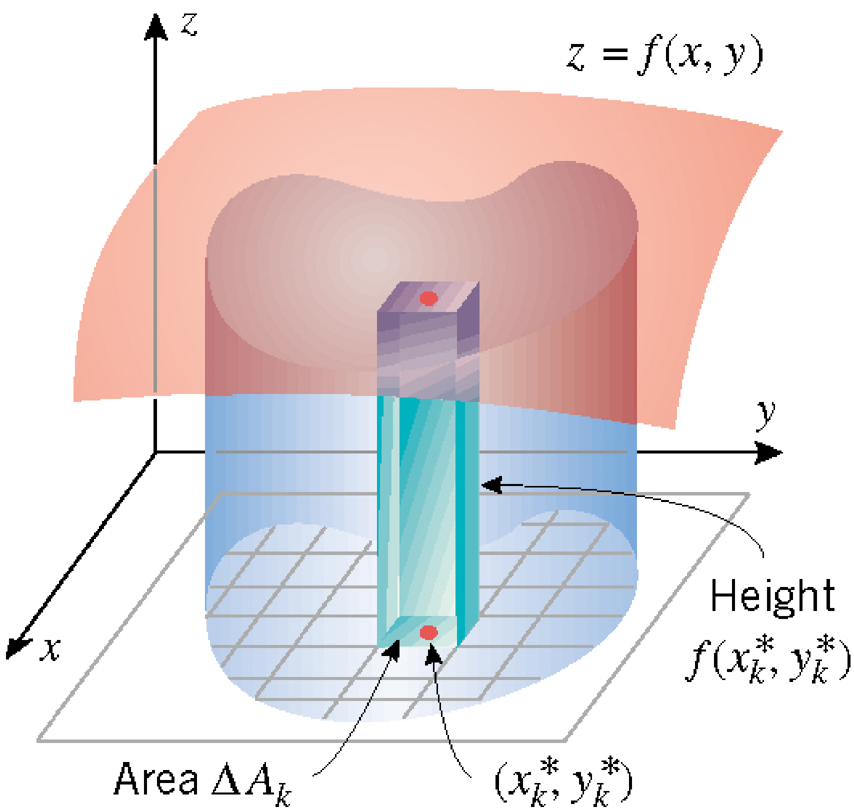
Volume under a surface $z = f(x, y) \geq 0$

- In Math 102, we did

  area problem ----------------→ definite integration of $f(x)$

- Here we see that

  volume problem ----------------→ double integration over a region R



The ***double integral*** of f over the rectangle R is

$$\iint_R f(x,y)\,dA = \lim_{m\to\infty, n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_j)\Delta A$$

if this limit exists.

This sum is called ***double Riemann sum***

The general notation of double integral is $\iint_R f(x,y)\,dA$, where R is the region in XY-plane and $dA = dxdy$ or $dA = dxdy$

If $f(x,y) \geq 0$ on R then the volume under $f(x,y)$ over the region R is given by

$$\iint_R f(x,y)\,dA$$

The integral $\iint_R 1 \cdot dA = \iint_R dA$ gives area of the region R

Basic properties of double integrals

- $\iint_R cf(x,y)\,dA = c\iint_R f(x,y)\,dA$
- $\iint_R [f(x,y) \pm g(x,y)]\,dA = \iint_R f(x,y)\,dA \pm \iint_R g(x,y)\,dA$
- $\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$ if $f(x,y) \geq g(x,y)$ on R
- $\iint_R f(x,y)\,dA = \iint_{R_1} f(x,y)\,dA + \iint_{R_2} f(x,y)\,dA$

  where R is divided into two regions R_1 and R_2.

Average Value of a Function

The average value of a function $f(x,y)$ defined on a rectangle R is

$$f_{ave} = \frac{1}{A(R)}\iint_R f(x,y)\,dA$$

Where $A(R)$ is the area of R.

Question 3(a)/988: Use a Riemann sum with $m = n = 2$ to estimate the value of $\iint_R \sin(x + y)dA$ where $R = [0,\pi]\times[0,\pi]$. Take the sample points to be lower left cornors.

Question 12/988: Evaluate the double integral $\iint_R (5 - x)dA$ over the region $R = \{(x, y): 0 \le x \le 5, \ \ 0 \le y \le 3\}$.

Question 18/1020: If $R = [0,1]\times[0,1]$. Show that $0 \le \iint_R \sin(x + y)dA \le 1$.