14.8 Lagrange Multipliers

Constrained Extrema for *f(x,y,z)* (with one constraint)

Also called finding constrained extrema

Question:

To optimize a function $f(x,y,z)$

subject to a given constraint $g(x,y,z)=k$

Assuming that extreme values exist and $\nabla g \neq 0$ on the surface $g(x,y,z)=k$

Answer: Lagrange theorem

If $f(x,y,z)$ has an extrema at (x_0,y_0,z_0) subject to $g(x,y,z)=k$ then $\nabla f(x_0,y_0,z_0)=\lambda \nabla g(x_0,y_0,z_0)$

λ is called Lagrange multiplier

Note

- The solutions of this equation gives all possible candidate for extreme points

Next

We learn how to use Lagrange theorem to solve constrained optimization problems

Solving Constrained Extrema Problems

Method of Lagrange Multipliers

- Given $f(x, y, z)$ and $g(x, y, z) = k$
- To find extrema of $f(x, y, z)$ subject to $g(x, y, z) = k$.

STEP 1: Find all values of x, y, z and λ by solving the equations

$$\begin{aligned} \nabla f(x, y, z) &= \lambda \nabla g(x, y, z) \\ g(x, y, z) &= k \end{aligned}$$

OR equivalent equations

$f_x = \lambda g_x$, $f_y = \lambda g_y$, $f_z = \lambda g_z$

and $g(x, y, z) = k$

- Four equations and four unknowns x, y, z, λ
- Can be solved to get x, y, z and λ

STEP 2: Evaluate f at all the points (x,y,z) obtained from Step 1. The largest of these values is the maximum value of f ; the smallest is the minimum value of f .

Question 10/971: Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2 y^2 z^2$ subject to constraint $x^2 + y^2 + z^2 = 1$.

Constrained Extrema for *f(x,y,z)* (with two constraints)

Question:

To optimize a function $f(x,y,z)$

subject to a given constraints

$g(x,y,z)=k$ and $h(x,y,z)=c$

Answer: Lagrange theorem

If $f(x,y,z)$ has an extrema at (x_0,y_0,z_0) subject to

$g(x,y,z)=k$ and $h(x,y,z)=c$ then

$$\nabla f(x_0,y_0,z_0)=\lambda\nabla g(x_0,y_0,z_0)+\mu\nabla h(x_0,y_0,z_0)$$

Solving Constrained Extrema Problems

Method of Lagrange Multipliers

- Given $f(x,y,z)$ and $g(x,y,z)=k$ and $h(x,y,z)=c$
- To find extrema of $f(x,y,z)$ subject to $g(x,y,z)=k$ and $h(x,y,z)=c$.

STEP 1: Find all values of x,y,z and λ and μ by solving the equations

$$\begin{aligned} \nabla f(x,y,z) &= \lambda \nabla g(x,y,z) \\ g(x,y,z) &= k \\ h(x,y,z) &= c \end{aligned}$$

OR equivalent equations

$f_x = \lambda g_x + \mu h_x$, $f_y = \lambda g_y + \mu h_y$, $f_z = \lambda g_z + \mu h_z$

$g(x,y,z)=k$ and $\mathrm{h}(x,y,z)=c$

- Four equations and four unknowns x, y, z, λ, μ
- Can be solved to get x, y, z, λ and μ

STEP 2: Evaluate f at all the points (x,y,z) obtained from Step 1. The largest of these values is the maximum value of f; the smallest is the minimum value of f.

Question 17/971: Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z)=yz+xy$ subject to constraint $xy=1$ and $y^2+z^2=1$.

Question 17/971: The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.