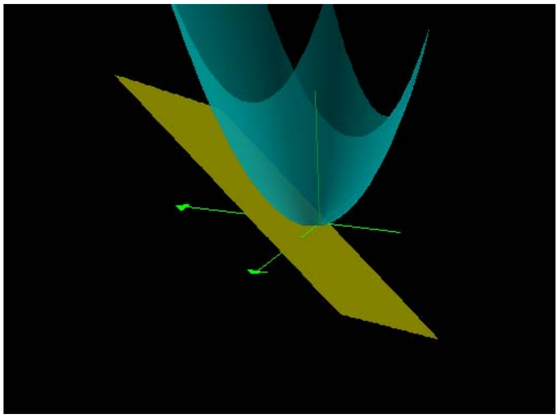
# 14.4 Tangent Planes Linear Approximations

## Tangent Planes

### What are tangent planes?



- Given the surface $z = f(x, y)$ and a point $P(x_0, y_0, z_0)$ on the surface.

Given the surface $z = f(x, y)$.

- The equation of the tangent plane at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

- In other words, an equation of the tangent plane to the graph of a function $f(x, y)$ of two variable at the point $(a, b, f(a, b))$ is

$$z = f(a,b) + f_x(a,b)(x - a) + f_y(a,b)(y - b)$$

$f$ has continuous partial derivatives

**Question 5/930**: Find an equation of the tangent plane to the surface $z = y\cos(x - y)$ at $(2, 2, 2)$.

## Linear Approximations of $z=f(x,y)$

The linear function whose graph is the tangent plane

$$L(x,y)=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

is called the ***linearization*** of $f(x,y)$ at $(a,b)$ and the

approximation

$$L(x,y)\approx f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

is called the ***linear approximation*** or the ***tangent plane approximation*** of $f(x,y)$ at $(a,b)$.

If $f(x,y)$ is differentiable at $(a,b)$ then $f(x,y)$

is approximately given by the linear function

$$L(x,y)=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

called linear approximation of $f$ at $(a,b)$

i.e. near $(a,b)$ we have $f(x,y)\approx L(x,y)$

Gives a good approximation only for $(x,y)$ near $(a,b)$

Both of above ideas can similarly be extended to functions of more variables

## Checking differentiability of *z=f(x,y)*

A function $f(x,y)$ is ***differentiable*** at the point $(a,b)$

if $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist near $(a,b)$ and are continuous at $(a,b)$.

**Question 15/930:** Explain why the function $f(x,y)=\tan^{-1}(x+2y)$ is differentiable at $(1,0)$. Then find the linearization $L(x,y)$ of the function at that point.

**Question 17/930:** Find the linear approximation of the function $f(x,y)=\sqrt{20-x^2-7y^2}$ at $(2,1)$ and use it to approximate $f(1.95,1.08)$.

# Differentials

- Total differential of $z = f(x, y)$

$$dz = f_x(x,y)dx + f_y(x,y)dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Also written as $df$

- Total differential of $w = f(x, y, z)$

$$dw = f_x dx + f_y dy + f_z dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

Also written as $df$

Let $\Delta x = x - x_0$, $\Delta y = y - y_0$ and $\Delta z = \Delta f = f(x, y) - f(x_0, y_0)$.

When $\Delta x = dx$ and $\Delta y = dy$ are small, we can approximate $\Delta z$ by $dz$.

**Question 27/931**: Find the differential $dw$ if $w = \ln\sqrt{x^2 + y^2 + z^2}$.

**Question 31/931**: The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.