14.1 Functions of Several Variables

Definition: Let $D \subseteq \mathbb{R}^2$. A function of two variables is a rule that assigns a unique real number $f(x,y)$ to each point $(x,y) \in D$.

The set D is called domain of $f(x,y)$.

Definition: Let $D \subseteq \mathbb{R}^3$. A function of three variables is a rule that assigns a unique real number $f(x,y,z)$ to each point $(x,y,z) \in D$.

The set D is called domain of $f(x,y,z)$.

Question 10/898: For $f(x,y,z) = \ln(25 - x^2 - y^2 - z^2)$

(a) Find $f(2,-2,4)$ **(b)** Find domain and range of $f(x,y,z)$

(c) Sketch domain of $f(x,y,z)$.

Question 16/898: Find and sketch domain of the function

$f(x,y) = \sqrt{y-x}\,\ln(y+x)$.

Exercise: Find domain of $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

Answer: $\{D = (x,y) \text{ such that } x+y+1 \geq 0 \;\&\; x \neq 1\}$

Graph of $f(x,y)$: The graph of $f(x,y)$ is the set of all points (x,y,z) satisfying $z = f(x,y)$

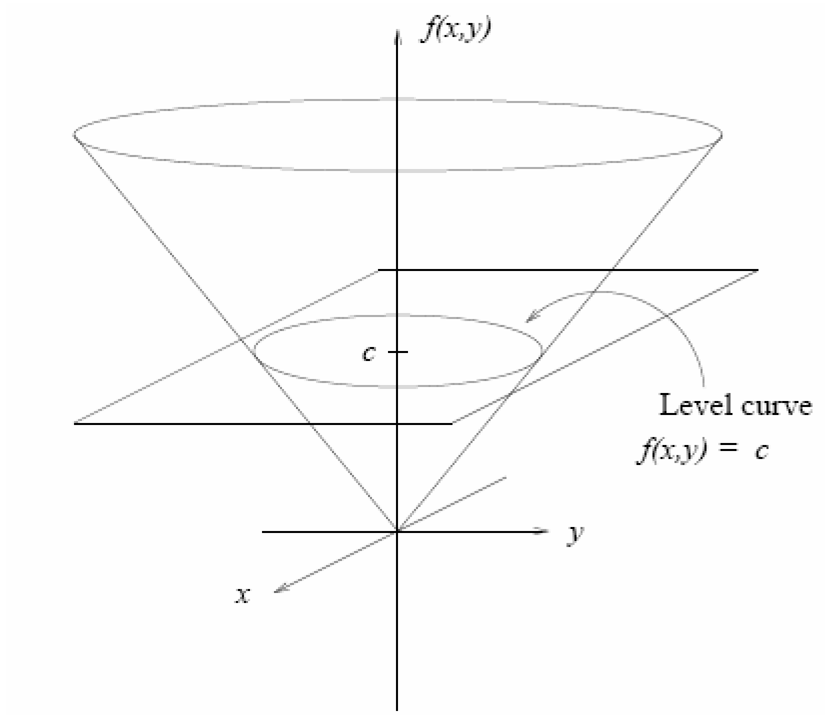
Similar to graph of $f(x)$

A surface lying above or below its domain

Question 28/899: Find domain of $f(x,y) = \sqrt{16 - x^2 - 16y^2}$ and sketch the graph of $f(x,y)$.

Level curves of $f(x, y)$:

Consider the function $f(x, y) = +\sqrt{x^2 + y^2}$. It represents the upper half of the elliptic cone.

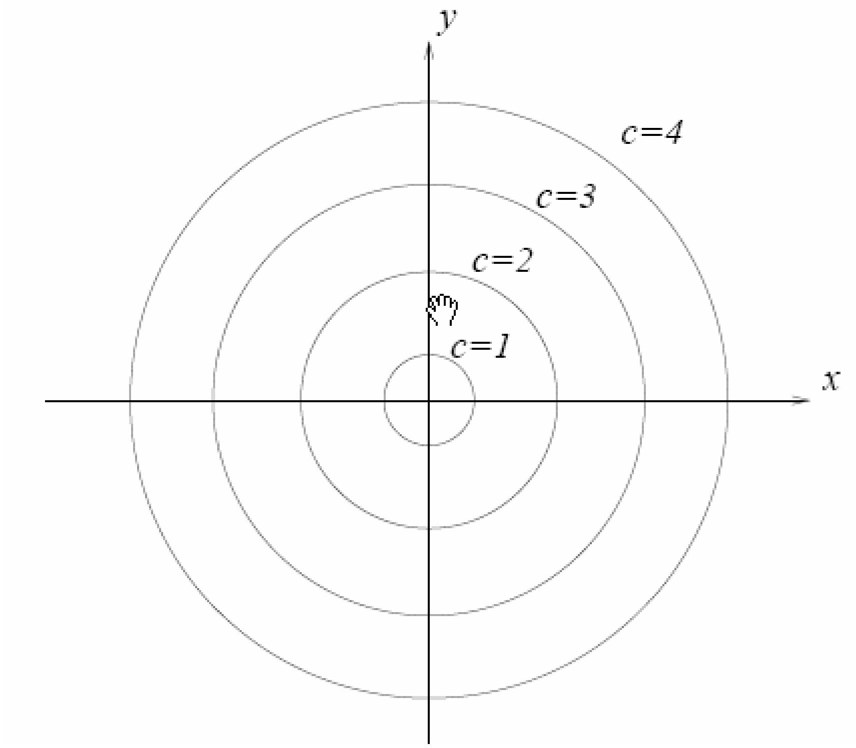


If we fix a value of $f(x, y)$, say $f(x, y) = c$, we are basically looking at the cross section of the surface. In this case, we have

$$+\sqrt{x^2 + y^2} = c \qquad \text{or} \qquad x^2 + y^2 = c^2$$

which is a circle of radius c.

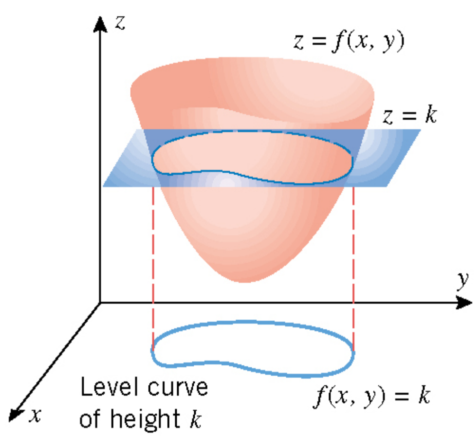
In this example, if we have different values of c, we will have cross sections. And we can plot them on the same graph.



We call these curves ***level curves.***

- The projection (on the XY plane) of the curve at height level ‘k’ is called a level curve with constant ‘k’, that is, the graph of the equation $f(x,y)=k$ (where k is any constant) is called level curve of height k.

- A set of some level curves of $f(x,y)$ is called a contour map (or contour plot) of $f(x,y)$.

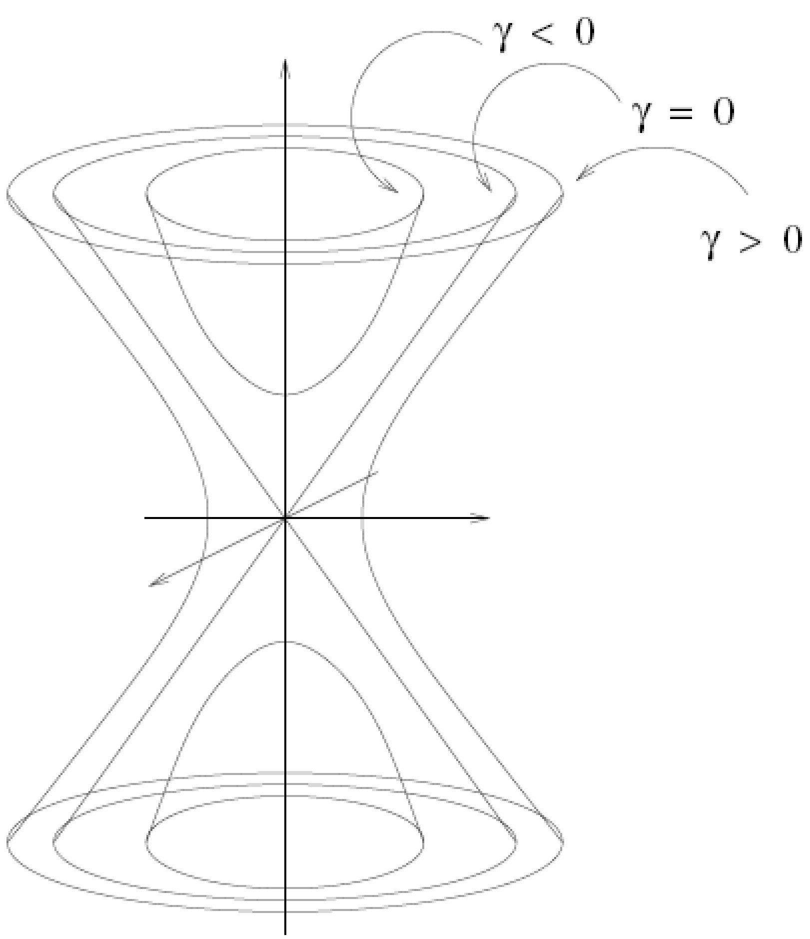


Question 42/931: Sketch the level curve (or cantor map) $z = k$ for $k = -2, -1, 0, 1, 2$ of the function $z = y \sec x$.

Level surfaces of $f(x, y, z)$

- Since the graph of $f(x, y, z)$ would be in 4 dimensional space, therefore, we cannot visualize it easily.
- But we can get an idea from its level surfaces.
- The graph of $f(x, y, z) = k$ is called a level surface of $f(x, y, z)$ with constant 'k'.

Example: Let $f(x, y, z) = x^2 + y^2 - z^2$. Then the level surfaces $f(x, y, z) = \gamma$ are given by



Question 60/900: Describe the level surfaces of the function

$f(x,y,z)=x^2+3y^2+5z^2$.

Similarly we can define

functions of n-variables for n>3

Exercise

For $f\left(x_1,x_2,\cdots,x_n\right)=\sum_{k=1}^{n}x_k$, find $f\left(1,1,\cdots,1\right)$