12.1 Three-Dimensional Coordinate Systems

3-Dimensional Rectangular Coordinate System

- **3-Dimensional Coordinate System**
  consists of three mutually perpendicular coordinate lines, called **X-axis, Y-axis, Z-axis** which intersect at a point called, *origin*.

- **Coordinate Planes**
  Three planes determined by coordinate axes are called **XY-plane, XZ-plane, YZ-plane**

In **XY-plane**, \( z = 0 \); In **YZ-plane**, \( x = 0 \); In **XZ-plane**, \( y = 0 \).

- **x = a** is a plane parallel to **YZ-plane** and intersecting **X-axis** at \( x = a \)
- **y = b** is a plane parallel to **XZ-plane** and intersecting **Y-axis** at \( y = b \)
- **z = c** is a plane parallel to **XY-plane** and intersecting **Z-axis** at \( z = c \)
• **Octants**
  Coordinate planes divide 3-space into 8 parts called octants

• **Coordinates**
  - Any point \( P \) can be determined by an ordered triplet \((x, y, z)\).
  - \( P \) has coordinates \((x, y, z)\) means
    
    To locate \( P \), we start from the origin, move \( x \)-units along X-axis, then \( y \)-units parallel to Y-axis and then \( z \)-units parallel to Z-axis.

**Some Basic Formulas in 3-Dimensional Coordinate System**

**Distance Formula**

The distance between points \( P_1(x_1, y_1, z_1) \) and \( P_2(x_2, y_2, z_2) \) is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
\]

**Question 8/797:** Find the lengths of the sides of the triangle with vertices
\( A (1, 2, -3) \), \( B (3, 4, -2) \) and \( C (3, -2, 1) \). Is \( ABC \) a right angle? Is it an isosceles triangle?
Midpoint Formula
The coordinates of midpoint of \( P_1(x_1, y_1, z_1) \) and \( P_2(x_2, y_2, z_2) \) are
\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)
\]

Sphere
Sphere with center \( (x_0, y_0, z_0) \) and radius \( r \) is given by
\[
(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2
\]

Need
- center
- origin

Given any equation of the form
\[
x^2 + y^2 + z^2 + Gx + Hy + Iz + J = 0.
\]
We can write it as
\[
(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = k
\]

- If \( k > 0 \) then sphere
- If \( k = 0 \) then only the point \( (x_0, y_0, z_0) \)
- If \( k < 0 \) then no graph

**Question 13/797**: Find an equation of the sphere that passes through the point \((4, 3, -1)\) and has center \((3, 8, 1)\).
**Question 18/797**: Show that the equation \(4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1\) represents a sphere, and find its center and radius.

**Question 22/797**: Find an equation of the largest sphere with center \((5, 4, 9)\) that is contained in the first octant.

**Question 30/797**: Describe in words the region of \(\mathbb{R}^3\) represented by the inequality \(1 \leq x^2 + y^2 + z^2 \leq 25\).