

## 12.1 Thre-Dimensional Coordinate Systems

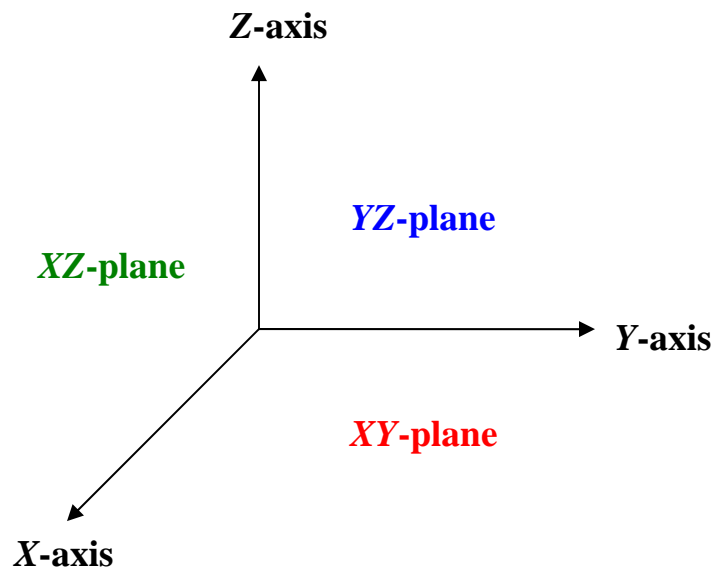
### 3-Dimensional Rectangular Coordinate System

- **3-Dimensional Coordinate System**

consists three mutually perpendicular coordinate lines, called **X-axis, Y-axis, Z-axis** which intersect at a point called, *origin*.

- **Coordinate Planes**

Three planes determined by coordinate axes are called **XY-plane, XZ-plane, YZ-plane**



In **XY-plane**,  $z=0$ ; In **YZ-plane**,  $x=0$ ; In **XZ-plane**,  $y=0$ .

- $x = a$  is a plane parallel to **YZ-plane** and intersecting **X-axis** at  $x = a$
- $y = b$  is a plane parallel to **XZ-plane** and intersecting **Y-axis** at  $y = b$
- $z = c$  is a plane parallel to **XY-plane** and intersecting **Z-axis** at  $z = c$

- **Octants**

Coordinate planes divide 3-space into 8 parts called octants

- **Coordinates**

- Any point  $P$  can be determined by an ordered triplet  $(x, y, z)$ .
- $P$  has coordinates  $(x, y, z)$  means

*To locate  $P$ , we start from the origin, move  $x$ -units along  $X$ -axis, then  $y$ -units parallel to  $Y$ -axis and then  $z$ -units parallel to  $Z$ -axis.*

### Some Basic Formulas in 3-Dimensional Coordinate System

#### Distance Formula

The distance between points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

**Question 8/797:** Find the lengths of the sides of the triangle with vertices  $A(1, 2, -3)$ ,  $B(3, 4, -2)$  and  $C(3, -2, 1)$ . Is  $ABC$  a right angle? Is it an isosceles triangle?

## Midpoint Formula

The coordinates of midpoint of  $P_1(x_1, y_1, z_1)$  and

$P_2(x_2, y_2, z_2)$  are

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

## Sphere

Sphere with center  $(x_0, y_0, z_0)$  and radius  $r$  is given

by  $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

Need

- center
- origin

Given any equation of the form

$$x^2 + y^2 + z^2 + Gx + Hy + Iz + J = 0.$$

by completing  
the square

We can write it as

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = k$$

If  $k > 0$   
then sphere

If  $k = 0$   
then only the point  
 $(x_0, y_0, z_0)$

If  $k < 0$   
then no graph

**Question 13/797:** Find an equation of the sphere that passes through the point  $(4, 3, -1)$  and has center  $(3, 8, 1)$ .

**Question 18/797**: Show that the equation  $4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$  represents a sphere, and find its center and radius.

**Question 22/797**: Find an equation of the largest sphere with center  $(5, 4, 9)$  that is contained in the first octant.

**Question 30/797**: Describe in words the region of  $\mathbb{R}^3$  represented by the inequality  $1 \leq x^2 + y^2 + z^2 \leq 25$ .