15.5 Triple Integrals

Triple integrals

Notation: $\iiint_G f(x,y,z)\,dV$

Where G is the solid and $dV = dxdydz$ or any other seven forms.

All rules are same as in case of double integral.

Evaluating Triple Integrals over Rectangular Boxes

Evaluated as iterated integrals

Let G be the rectangular box $a \le x \le b$, $c \le y \le d$, $k \le z \le l$ then

$$\iiint_G f(x,y,z)\,dV = \int_a^b \int_c^d \int_k^l f(x,y,z)\,dzdydx$$

- Or any other ordering with proper adjustment of limits of integration.

Question 7/1054: Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dzdydx$.

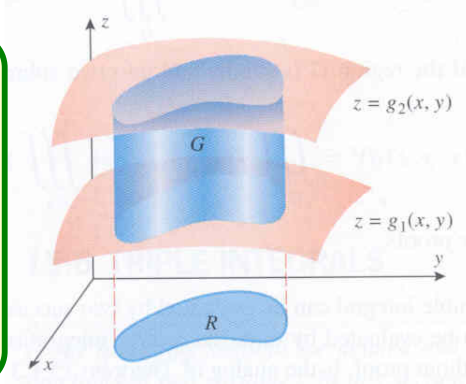
Evaluating Triple Integrals over Simple XY-region

Simple XY-region

A region G given by

- $G=\{(x,y,z):(x,y)\in R \text{ and } g_1(x,y)\le z\le g_2(x,y)\}$

where R is projection of G on XY-plane



Evaluation of triple integral over such regions as

- $$\iiint_G f(x,y,z)dV=\iint_R\left[\int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)dz\right]dA$$

Evaluating Triple Integrals over Simple YZ-region

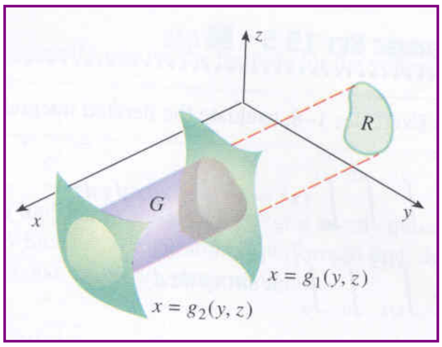
See class explanation

Simple YZ-region

A region G given by

- $G=\{(x,y,z):(y,z)\in R \text{ and } g_1(y,z)\le x\le g_2(y,z)\}$

where R is projection of G on YZ-plane



Evaluation of triple integral over such regions as

- $$\iiint_G f(x,y,z)dV=\iint_R\left[\int_{g_1(y,z)}^{g_2(y,z)} f(x,y,z)dx\right]dA$$

Evaluating Triple Integrals over Simple XZ-region

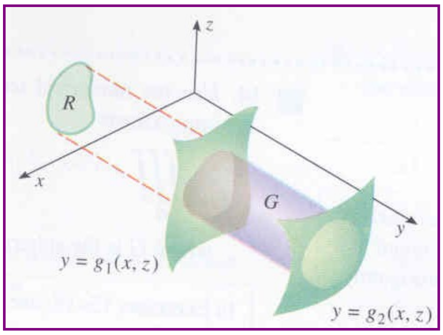
See class explanation

Simple XZ-region

A region G given by

- $G = \{(x,y,z) : (x,z) \in R \text{ and } g_1(x,z) \le y \le g_2(x,z)\}$

where R is projection of G on XZ-plane



Evaluation of triple integral over such regions as

- $$\iiint_G f(x,y,z)\,dV = \iint_R \left[\int_{g_1(x,z)}^{g_2(x,z)} f(x,y,z)\,dy \right] dA$$

Question 11/1054: Evaluate $\iiint_G xyz\,dV$, where G is the solid in the first octant that is bounded by the parabolic cylinder $z = 2 - x^2$ and the planes $z = 0$, $y = x$, and $y = 0$.

- Try to project the region G on XY, YZ, XZ planes
- and see which gives you easier calculations and better visualization of G & R.
- **Easiest:** thinking as integral over XY-region

Volume as Triple Integral

The volume of a 3-dimensional region G is given by

$$V = \iiint_G dV$$

Question 17/1054: Use triple integral to find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4$ and $z = 0$.

Question 32(c)/1055: Reverse the order in the form $dzdydx$.

$$\int_0^4 \int_0^{4-y} \int_0^{\sqrt{z}} f(x, y, z)\, dxdzdy \,.$$

Solve all the solved Examples given in the book and Questions 1—12, 15—18, 21—22 and 31—32.