15.1 Double integrals

Motivation:

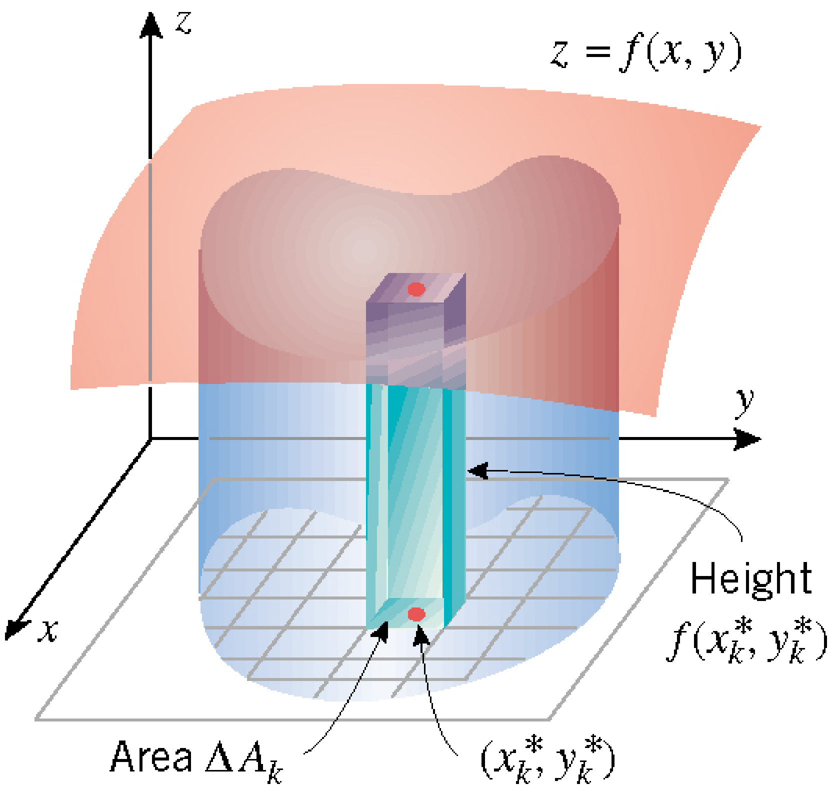
Volume under a surface $z = f(x, y) \geq 0$

- In Math 102, we did

  area problem ----------------→ definite integration of *f(x)*

- Here we see that

  volume problem ----------------→ double integration over a region *R*



Double Integrals & Basic Properties

The general notation of double integral is $\iint_R f(x, y)dA$, where R is the region in XY-plane and $dA = dxdy$ or $dA = dxdy$

If $f(x,y) \geq 0$ on R then the volume under $f(x,y)$ over the region R is given by

$$\iint_R f(x,y)dA$$

The integral $\iint_R 1 \cdot dA = \iint_R dA$ gives area of the region R

Basic properties of double integrals

- $\iint_R cf(x,y)dA = c\iint_R f(x,y)dA$
- $\iint_R [f(x,y) \pm g(x,y)]dA = \iint_R f(x,y)dA \pm \iint_R g(x,y)dA$
- $\iint_R f(x,y)dA = \iint_{R_1} f(x,y)dA + \iint_{R_2} f(x,y)dA$

  where R is divided into two regions R_1 and R_2.

Main Question:

Efficient methods of evaluating double integrals

Evaluating Double Integrals over Rectangular Regions

more general regions in next section

Partial Integration

$\int_a^b f(x, y)\,dx$ = partial definite integral w.r.t. x.

(It is evaluated by holding y fixed and integrating w.r.t x)

$\int_c^d f(x, y)\,dy$ = partial definite integral w.r.t. y.

(It is evaluated by holding x fixed and integrating w.r.t y)

If the region R is defined by $a \le x \le b$, $c \le y \le d$

then

$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

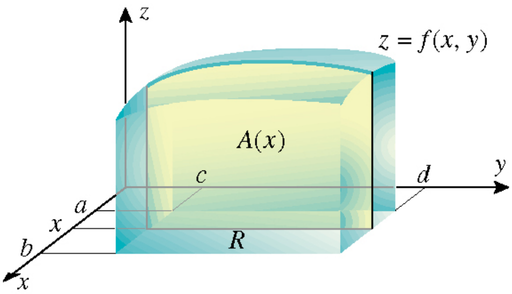
i.e. R is rectangular region $[a,b] \times [c,d]$

Note:

- inner limits for inner differential
- outer limits for outer differential

These integrals are evaluated as iterated integrals

- $\int_c^d \int_a^b f(x,y)\,dx\,dy = \int_c^d \left[\int_a^b f(x,y)\,dx \right] dy$
- $\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_a^b \left[\int_c^d f(x,y)\,dy \right] dx$



Question 11/1019: Evaluate the integral $\int_0^{\ln 2}\int_0^1 xye^{y^2x}\,dydx$.

Question 15/1020: Evaluate the double integral $\iint_R x\sqrt{1-x^2}\,dA$ over the region $R=\{(x,y):0\le x\le 1,\ \ 2\le y\le 3\}$.

Question 22/1020: Use the double integral to find the volume in the first octant bounded by the coordinate planes, the plane $y=4$, and the plane $(x/3)+(z/5)=1$.

Solve all solved examples given in the book and Questions 1—16 and 19—25.