14.9 Lagrange Multipliers

Constrained Extrema for *f(x,y)* (with one constraint)

Also called finding constrained extrema

Question:

To optimize a function $f(x,y)$

subject to a given constraint $g(x,y)=c$

e.g. the constraint can be boundary.

Answer: Lagrange theorem

If $f(x,y)$ has an extrema at (x_0,y_0) subject to $g(x,y)=c$ then $\nabla f(x_0,y_0)=\lambda\nabla g(x_0,y_0)$

λ is called Lagrange multiplier

Note

- The solutions of this equation gives all possible candidate for extreme points

Next

We learn how to use Lagrange theorem to solve constrained optimization problems

Solving Constrained Extrema Problems

Method of Lagrange Multipliers

- Given $f(x,y)$ and $g(x,y)=c$
- To find extrema of $f(x,y)$ subject to $g(x,y)=c$.

STEP 1: Solve the equations

$$\begin{aligned} \nabla f(x,y) &= \lambda \nabla g(x,y) \\ g(x,y) &= c \end{aligned}$$

OR equivalent equations

$$\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ g(x,y) &= c \end{aligned}$$

- Three equations and three unknowns $x,\ y,\ \lambda$
- Can be solved to get $x,\ y$ and λ

STEP 2: Examine the points (x,y) obtained from Step 1 for extrema

Question 11/1009: Find the extrema of $f(x,y,z)=xyz$ subject to constraint $x^2+y^2+z^2=1$.

Question 17/1009: Find the point on the circle $x^2+y^2=45$ that is closest to and farthest from (1,2).

Solve all the solved Examples given in the book and Questions 5—20.