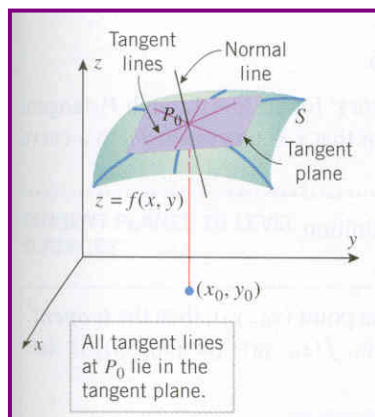


14.7 Tangent Planes and Normal Vectors

What are tangent planes & normal lines?



Equation of Tangent Plane & Normal Line to a Surface

- Given a surface $z = f(x, y)$, we can write it as $F(x, y, z) = c$.
 - ❖ i.e. we can think of the surface $z = f(x, y)$ as a level surface $F(x, y, z) = c$ of a function of three variables.
 - ❖ Hence, $\nabla F(x_0, y_0, z_0)$ will be normal to the surface $F(x, y, z) = c$ or $z = f(x, y)$ at (x_0, y_0, z_0) .

See
section
14.6

Given the surface $F(x, y, z) = c$.

- The vector $\nabla F(x_0, y_0, z_0)$ is normal to surface at (x_0, y_0, z_0) .
- The equation of the tangent plane at (x_0, y_0, z_0) is

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

- The equation of the normal line at (x_0, y_0, z_0) is

$$x = x_0 + F_x(x_0, y_0, z_0)t, \quad y = y_0 + F_y(x_0, y_0, z_0)t, \quad z = z_0 + F_z(x_0, y_0, z_0)t$$

What do
we need for
equation of
plane and
equation of
line ?

Question 7/990: Let $x^2 + y^2 + z^2 = 25$ be a surface.

- (a) Find the normal vector at $P(-3, 0, 4)$.
- (b) Find an equation for the tangent plane at $P(-3, 0, 4)$.
- (c) Find the parametric equations for the normal line to the surface at $P(-3, 0, 4)$.

Question 11/990: Find a point on the surface $z = 8 - 3x^2 - 2y^2$ at which the tangent plane is perpendicular to the line $x = 2 - 3t$, $y = 7 + 8t$, $z = 5 - t$.

Exercise: Find the equation of tangent plane and normal line to the surface

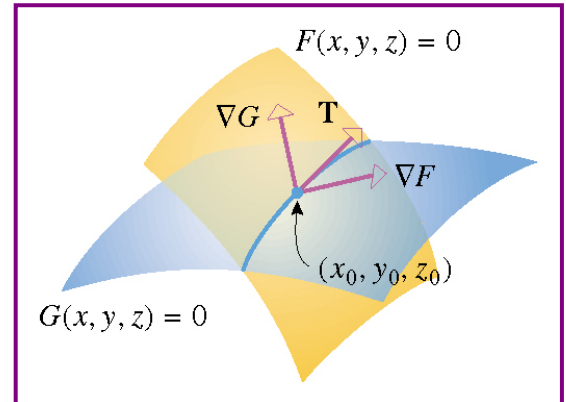
$$x = y^2 + z^2 - 2 \text{ at } (-1, 1, 0).$$

Note: to write Eq. in form $F(x, y, z) = c$

Using gradient to find tangent lines to curves of intersection of two surfaces

Let $F(x, y, z) = 0$, $G(x, y, z) = 0$ be two intersecting surfaces & (x_0, y_0, z_0) be a point on the curve of intersection.

- Then both $\nabla F(x_0, y_0, z_0)$ and $\nabla G(x_0, y_0, z_0)$ are normal to the curve of intersection.
- This implies $\vec{v} = \nabla F \times \nabla G$ is parallel to the tangent line to the curve of intersection.
- Hence, we can write equation of the tangent line to the curve of intersection.



Example 25/990: Find the parametric equations of tangent line to the curve of intersection of $x^2 + z^2 = 25$ and $y^2 + z^2 = 25$ at $P(3, -3, 4)$.

Solve all the Solved Examples given in the book and Questions 1—25.