14.4 *Differentiability, Local Linearity and Differentials*

Checking differentiability of *z=f(x,y)*

A function $f(x,y)$ is differentiable at the point (x_0, y_0)

if $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ are continuous at (x_0, y_0).

Local linear approximation of *z=f(x,y)*

If $f(x,y)$ is differentiable at (x_0, y_0) then $f(x,y)$

is approximately given by the linear function

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Called local linear approximation of f at (x_0, y_0)

- i.e. near (x_0, y_0) we have $f(x,y) \approx L(x,y)$

Gives a good approximation only for (x,y) near (x_0, y_0)

Both of above ideas can similarly be extended to functions of more variables

Question 1(a)/961: Show that $f(x,y) = e^x \sin y$ is differentiable at $(0,0)$.

(b) Find local linear approximation of $f(x,y) = e^x \sin y$ at $(0,0)$.

(c) Use this local linear approximation to approximate $f(0.1, -0.9)$.

Question 11/961: Suppose that a function $f(x, y, z)$ is differentiable at the point $(3,2,1)$ and $L(x, y, z) = x - y + 2z - 2$ is the local linear approximation to f at $(3,2,1)$. Find $f(3,2,1)$, $f_x(3,2,1)$, $f_y(3,2,1)$ and $f_z(3,2,1)$.

Question 15/961: Given a function $f(x, y, z) = xy + z^2$ along with its local linear approximation $L(x, y, z) = y + 2z - 1$ at a point P. Determine the point P.

Question 20/961: Find the local linear approximation L of the function $f(x, y) = \ln xy$ at $P(1,2)$ and $Q(1.01, 2.02)$. Also compare the error in approximating f by L at Q with the distance between P and Q.

Total differential

- Total differential of $z = f(x,y)$ at (x_0, y_0)

$$dz = f_x(x_0,y_0)dx + f_y(x_0,y_0)dy$$
$$= \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} dx + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} dy$$

Also written as df

- Total differential of $w = f(x,y,z)$

$$dw = f_x dx + f_y dy + f_z dz$$
$$= \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

Also written as df

Let $\Delta x = x - x_0$, $\Delta y = y - y_0$ and $\Delta z = \Delta f = f(x,y) - f(x_0,y_0)$.

When $\Delta x = dx$ and $\Delta y = dy$ are small, we can approximate Δz by dz.

Question 30/962: Compute the differential dz if $z = \sec^2(x-3y)$.

Question 41/962: Use differential to approximate the change

in $f(x,y,z) = 2xy^2z^3$ as (x,y,z) varies from $P(1,-1,2)$ to $Q(0.99,-1.02,2.02)$.

Question 46/962: The radius and height of a right circular cone are measured with errors of at most 1% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume.

Question 53/962: The area of a triangle is to be computed from the formula $A = (1/2)ab\sin\theta$, where a and b are the length of two sides and θ is the included angle. Suppose that a, b and θ are measured to be 40ft, 50ft, and 30°, respectively. Use differential to approximate the maximum error in the calculated value of A if the maximum errors in a, b and θ are 1/2ft, 1/4ft and 2°, respectively.

Solve all solved examples given in the book and Questions 1—54.