14.1 Functions of Two or More Variables

Definition: Let $D \subseteq \mathbb{R}^2$. A function of two variables is a rule that assigns a unique real number $f(x,y)$ to each point $(x,y) \in D$.

The set D is called domain of $f(x,y)$.

Definition: Let $D \subseteq \mathbb{R}^3$. A function of three variables is a rule that assigns a unique real number $f(x,y,z)$ to each point $(x,y,z) \in D$.

The set D is called domain of $f(x,y,z)$.

Question 21/930: For $f(x,y) = \dfrac{1}{x - y^2}$

(a) Find $f(3,2)$ **(b)** Find domain of $f(x,y)$ **(c)** Sketch domain of $f(x,y)$

Exercise: Find domain of $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

Answer: $\{D = (x,y) \text{such that } x + y + 1 \geq 0 \ \& \ x \neq 1\}$

Graph of $f(x,y)$: The graph of $f(x,y)$ is the set of all points (x,y,z) satisfying $z = f(x,y)$

Similar to graph of $f(x)$

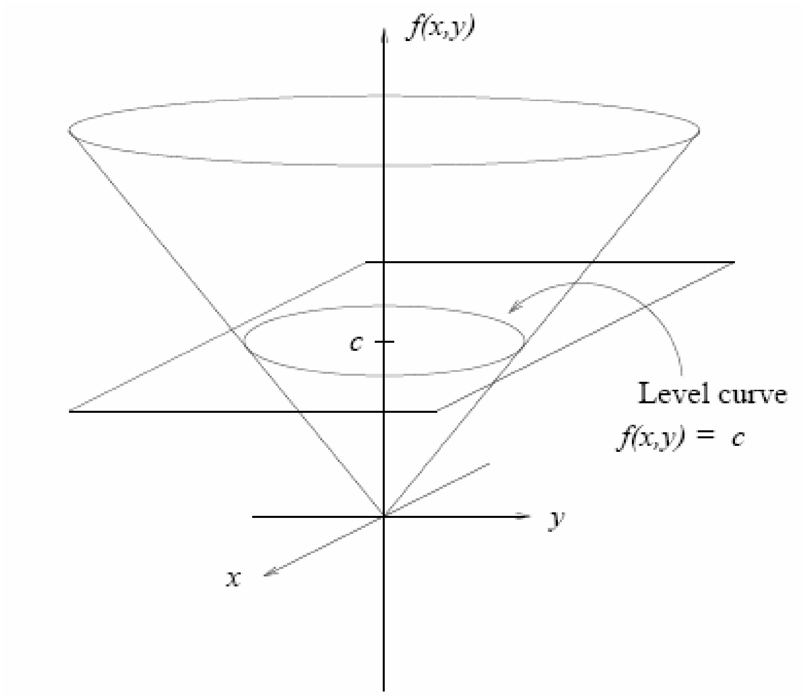
A surface lying above or below its domain

Question 26/930: Find the domain of $f(x,y)=\sqrt{9-x^2-y^2}$ and sketch the graph of $f(x,y)$.

Question24©/930: Find domain of $f(x,y,z)=\dfrac{xyz}{x+y+z}$. Find $f\left(0,\sqrt{3},-\sqrt{2}\right)$.

Level curves of $f(x,y)$:

Consider the function $f(x,y)=+\sqrt{x^2+y^2}$. It represents the upper half of the elliptic cone.

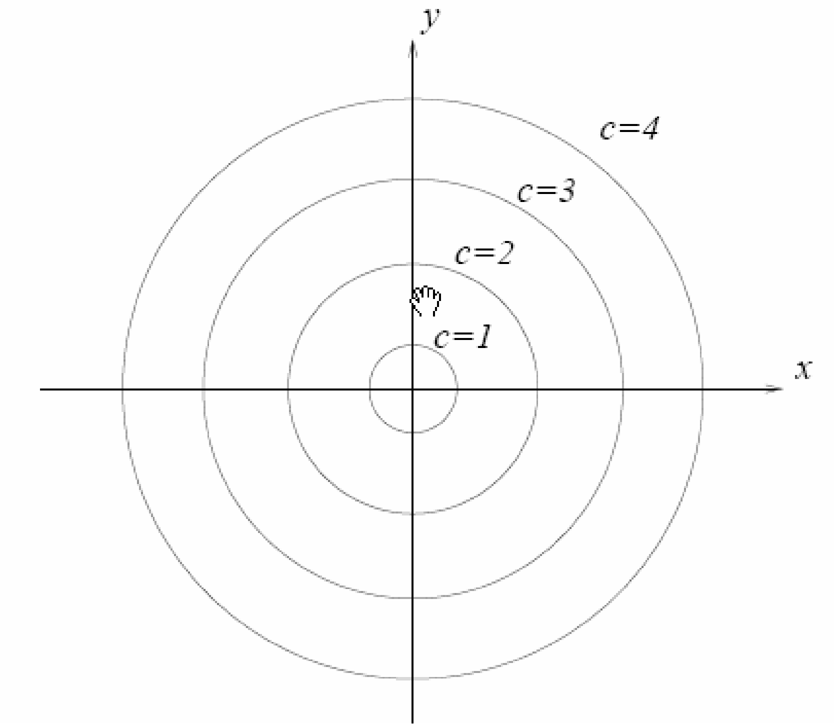


If we fix a value of $f(x,y)$, say $f(x,y)=c$, we are basically looking at the cross section of the surface. In this case, we have

$$+\sqrt{x^2+y^2}=c \qquad \text{or} \qquad x^2+y^2=c^2$$

which is a circle of radius c.

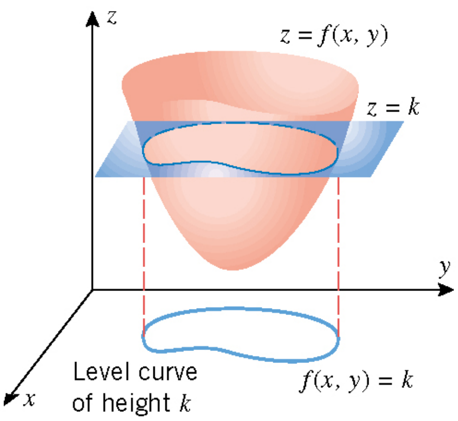
In this example, if we have different values of c, we will have cross sections. And we can plot them on the same graph.



We call these curves ***level curves.***

- The projection (on the XY plane) of the curve at height level ‘k’ is called a level curve with constant ‘k’, that is, the graph of the equation $f(x,y)=k$ (where k is any constant) is called level curve of height k.

- A set of some level curves of $f(x,y)$ is called a contour map (or contour plot) of $f(x,y)$.

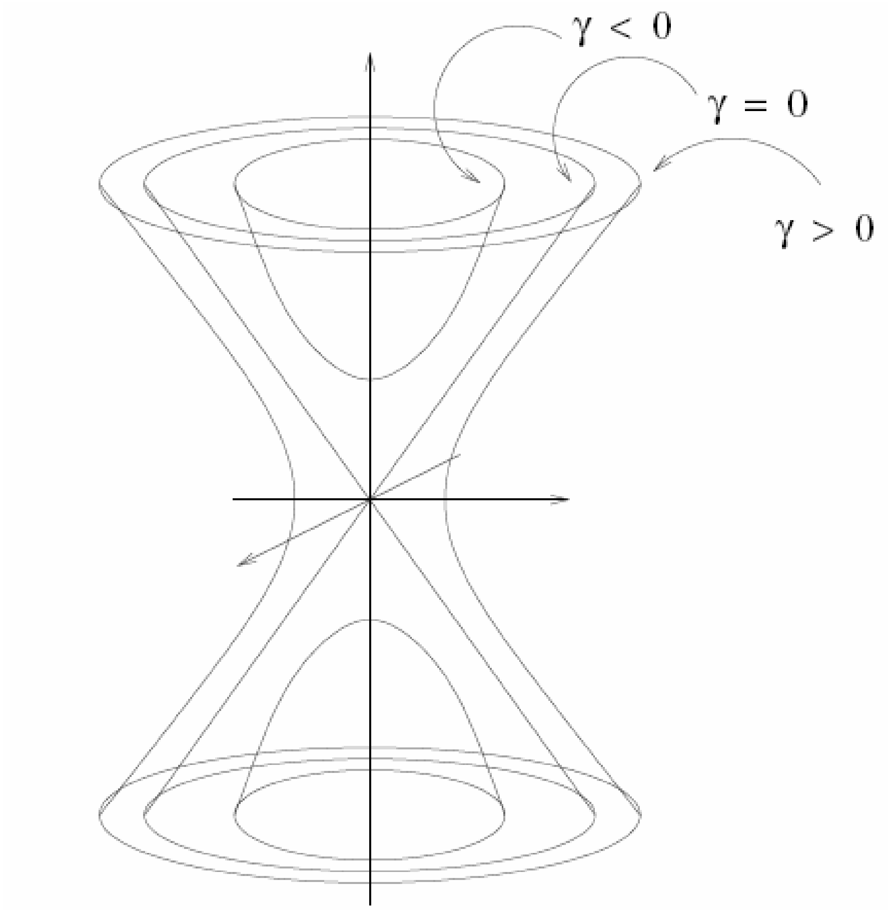


Question 41/931: Sketch the level curve $z = k$ for $k = -2, -1, 0, 1, 2$ of the function $z = x^2 + y$.

Level surfaces of $f(x, y, z)$

- Since the graph of $f(x, y, z)$ would be in 4 dimensional space, therefore, we cannot visualize it easily.
- But we can get an idea from its level surfaces.
- The graph of $f(x, y, z) = k$ is called a level surface of $f(x, y, z)$ with constant 'k'.

Example: Let $f(x, y, z) = x^2 + y^2 - z^2$. Then the level surfaces $f(x, y, z) = \gamma$ are given by



Question 47/932: Sketch the level surfaces of $f(x,y,z)=k$.

$f(x,y,z)=z-x^2-y^2+4; \qquad k=7$.

Question 55/932. Let $f(x,y,z)=x^2+y^2-z$. Find an equation of the level surface that passes through the point (a) $(1,-2,0)$ (b) $(1,0,3)$.

Similarly we can define

functions of n-variables for n>3

Exercise For $f\left(x_1,x_2,\cdots,x_n\right)=\sum_{k=1}^{n}x_k$, find $f\left(1,1,\cdots,1\right)$

Solve all Solved examples given in the book and Questions 1—8, 13—34, 39—56.