12.8 Cylindrical and Spherical Coordinates

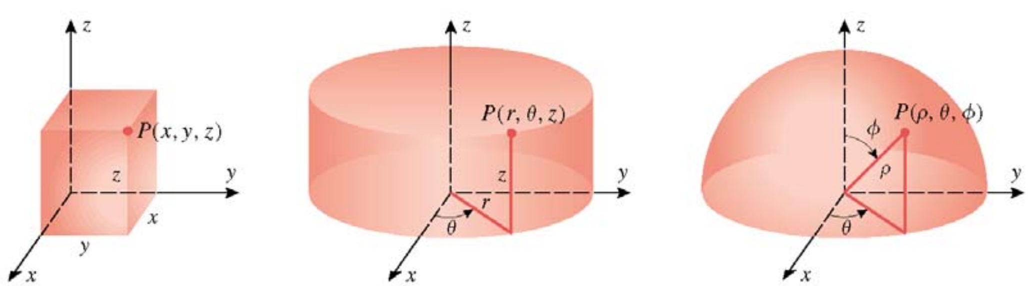
- In 2-dimensions, we learnt polar coordinates which gave an easier description of some curves.
- Here, we introduce two coordinate systems in 3-dimensions, known as cylindrical coordinate system and spherical coordinate system which give easier description of some surfaces.

In 2-space, every point can be represented by two ways: one is in rectangular coordinate system and other is in polar coordinate system. In 3-space each point can be represented by three ways:

(i) *rectangular coordinates*

(ii) *cylindrical coordinates and*

(iii) *spherical coordinates.*



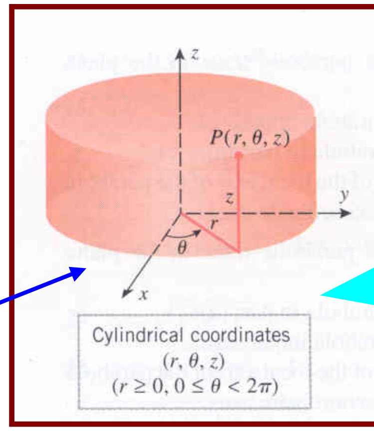
Rectangular coordinates (x, y, z) — (a)

Cylindrical coordinates (r, θ, z) $(r \geq 0, 0 \leq \theta < 2\pi)$ — (b)

Spherical coordinates (ρ, θ, ϕ) $(\rho \geq 0, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi)$ — (c)

What are cylindrical coordinates?

A point P in 3-space represented by coordinates (r,θ,z), where r, θ and z are as shown in the figure



Restriction:
$r \geq 0$ and
$0 \leq \theta < 2\pi$

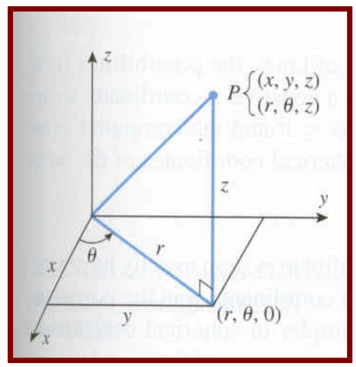
Relation between cylindrical & rectangular coordinates

- Cylindrical to rectangular: $(r,\theta,z) \to (x,y,z)$

$$x = r\cos\theta,\quad y = r\sin\theta,\quad z = z$$

- Rectangular to cylindrical: $(x,y,z) \to (r,\theta,z)$

$$r = \sqrt{x^2 + y^2},\quad \tan\theta = \frac{y}{x},\quad z = z$$



Question 2(a)/854: Convert the rectangular coordinates $\left(\sqrt{2},-\sqrt{2},0\right)$ to the cylindrical coordinates.

Question 5(d)/854: Convert the cylindrical coordinates $\left(4,\pi/2,-1\right)$ into cylindrical coordinates.

Question 42/855 Find the equation of ellipsoid $x^2 + y^2 + z^2 = 2z$ in cylindrical coordinates.

Exercise: Identify the following surfaces (which given in cylindrical coordinates)

Hint: First convert into rectangular coordinates.

(a) $r = 5$

(b) $r^2 + z^2 = 100$

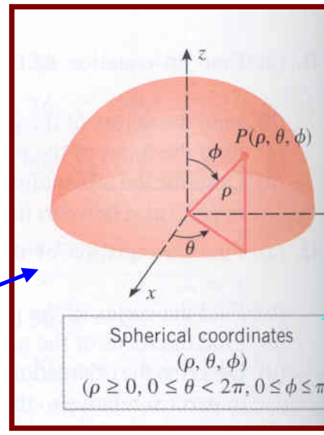
(c) $z = r$

What are spherical coordinates?

A point P in 3-space represented by coordinates (ρ,θ,ϕ), where

- ρ is distance of P from origin
- θ, ϕ as shown in the figure

See class explanation



Restriction:
$\rho \geq 0$ and
$0 \leq \theta < 2\pi$
$0 \leq \phi \leq \pi$

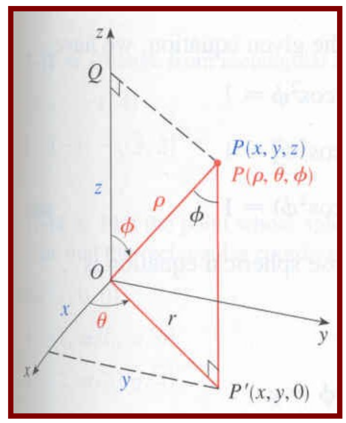
Relation between spherical & rectangular coordinates

- Spherical to rectangular: $(\rho,\theta,\phi) \to (x,y,z)$

$$x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi$$

- Rectangular to spherical: $(x,y,z) \to (\rho,\theta,\phi)$

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \tan\theta = \frac{y}{x}, \quad \cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$



Question 5(c)&8(d)/854:

(a) Convert $(0, 3\sqrt{3}, 3)$ from rectangular to spherical coordinates.

(b) Convert $\left(4, \pi/2, \pi/3\right)$ from spherical to rectangular coordinates.

Question 40/855: Find the equation of $x^2 + y^2 - z^2 = 1$ (of hyperboloid of one sheet) in spherical coordinates

Exercise: Identify the following surfaces

(a) $\rho = 5$ **(b)** $\rho\sin\phi = 2$

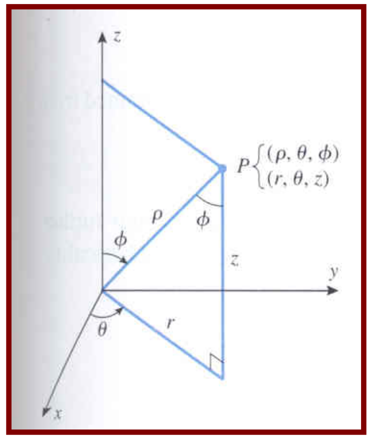
Conversion between cylindrical & spherical coordinates

- **Spherical to cylindrical:** $(\rho,\theta,\phi)\rightarrow(r,\theta,z)$

$$r=\rho\sin\phi,\quad \theta=\theta,\quad z=\rho\cos\phi$$

- **Cylindrical to Spherical:** $(r,\theta,z)\rightarrow(\rho,\theta,\phi)$

$$\rho=\sqrt{r^2+z^2},\ \theta=\theta,\ \tan\phi=\frac{r}{z}$$



Question 9(a)/854: Convert $\left(\sqrt{3},\frac{\pi}{6},3\right)$ from cylindrical to spherical coordinates.

Question 12(c)/854: Convert $\left(0,\frac{3\pi}{4},-\sqrt{2}\right)$ from spherical to cylindrical coordinates.

Solve all solved Examples given in the book and Questions 1-12,15-42.