Some Formulas (Chapter 11)

1. The general form of the polar coordinates is (r, θ).
2. The general form of the polar equations is $r = f(\theta)$.
3. $(-r, \theta)$ and $(r, \theta + \pi)$ are polar coordinates of the same points.
4. To convert from polar coordinates to rectangular coordinates: $x = r\cos\theta$ and $y = r\sin\theta$.
5. To convert from rectangular coordinates to polar coordinates: $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^1(y/x)$.
6. For parametric equations $x = f(t)$ and $y = g(t)$,

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad y'' = \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}, \quad y''' = \frac{d^3y}{dx^3} = \frac{dy''/dt}{dx/dt}.$$

   (a) The tangent to a parametric curve $x = f(t)$, $y = g(t)$ is horizontal at the points where $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$;
   (b) The tangent to a parametric curve $x = f(t)$, $y = g(t)$ is vertical at the points where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$;
   (c) The points where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$ are called singular points.

7. For polar equation $r = f(\theta)$,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r\cos\theta + \sin\theta\frac{dr}{d\theta}}{-r\sin\theta + \cos\theta\frac{dr}{d\theta}}.$$

   (a) The tangent to a polar curve $r = f(\theta)$ is horizontal at the points where $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} \neq 0$;
   (b) The tangent to a polar curve $r = f(\theta)$ is vertical at the points where $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$;
   (c) The points where $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} = 0$ are called singular points.

8. The arc length L of a polar curve from $\theta = \alpha$ to $\theta = \beta$ is

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

9. The area of the surface S generated by revolving the parametric curve $x = f(t)$, $y = g(t)$, $(a \leq t \leq b)$

   (a) about the x-axis is

$$S = \int_a^b 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt;$$

   (b) about the y-axis is

$$S = \int_a^b 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

10. The area A of the region enclosed by the polar curve $r = f(\theta)$ and the rays $\theta = \alpha$ and $\theta = \beta$ is

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 d\theta = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta,$$

where $\alpha < \beta \leq \alpha + 2\pi$, f is continuous and $f(\theta) \neq 0$ for $\alpha < \theta < \beta$.

11. The area of the surface S generated by revolving the potion of the polar curve $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$

   (a) about the polar axis (about $\theta = 0$) is

$$S = \int_{\alpha}^{\beta} 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta;$$

   (b) about $\theta = \pi/2$ is

$$S = \int_{\alpha}^{\beta} 2\pi r \cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta;$$