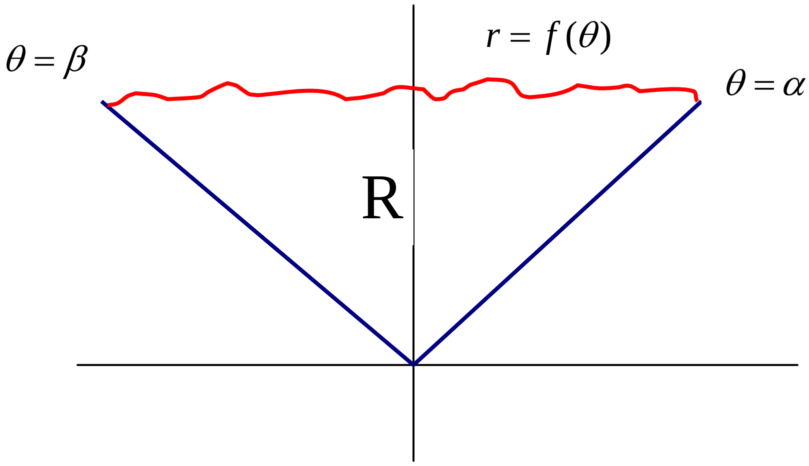
11.3 Area in Polar Coordinates



The area A of the region R enclosed by the polar curve $r = f(\theta)$ from the rays $\theta = \alpha$ to $\theta = \beta$ is

$$A = \int_{\alpha}^{\beta} \frac{1}{2}[f(\theta)]^2 \, d\theta = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$$

Assume that α and β satisfying $\alpha < \beta \le \alpha + 2\pi$, $f(\theta)$ is continuous and $f(\theta) \ne 0$ for $\alpha < \theta < \beta$

Procedure for finding the area:

- Sketch the curve
- Find α and β (See the explanation in class)
- Set up and evaluate integral

Caution About Finding Points of Intersections of Polar Curves

Equating two polar curves does not always give all points of intersection.

So the best strategy to find all points of intersections is

- to equate the equations and solve
- also to sketch the curve to see if there are more intersection points.

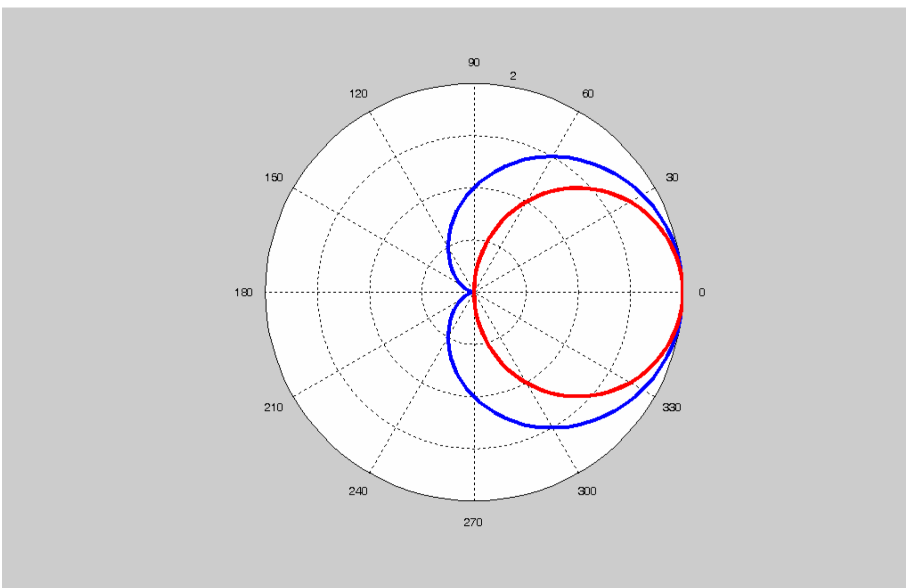
Why?

Because a point may have different representation as polar coordinates

Question: Find all points of intersections of $r = 1 + \cos\theta$ and $r = 2\cos\theta$ for $0 \le \theta < 2\pi$.

Solution:

- By equating $1 + \cos\theta = \cos 2\theta$ we get $\cos\theta = 1 \quad \Rightarrow \quad \theta = 0$.
- From the sketch below, we see that "pole" is also a point of intersection.



Question 6/747 Find the area of the region in the first quadrant with the cardioid $r = 1 + \sin\theta$.

Question 9/747 Find the area of the region enclosed by the inner loop of the lemacon $r = 1 + 2\cos\theta$.

Question 14/748: Find the area of the region inside both cardioid $r = 1 + \cos\theta$ and circle $r = 3\cos\theta$.

Question 21/748: Find the area of the region inside the circle $r = 10$ and to the right of the line $r = 6\sec\theta$.

Question: Find the area of the region bounded by the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

Surface Generated by Revolving the Polar Curve

The area of the surface generated by revolving the potion of the polar curve $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the polar axis is

$$S = \int_{\alpha}^{\beta} 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

and the area of the surface generated by revolving the potion of the polar curve $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the line $\theta = \pi/2$ is

$$S = \int_{\alpha}^{\beta} 2\pi r \cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Question 28/748 Find the area of the surface generated by revolving the spiral $r = e^{\theta}$ $(0 \le \theta \le \pi/2)$ about the line $\theta = \pi/2$.

Solve all the solved questions given in the book and Questions

1—22, 24—25, 27—30.