King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

MATH - 201 Final Examination (053)

Venue: Bldg. 54
Time: 7:00 p.m. to 10:00 p.m.
Date: August 15, 2006
Max. Marks: 80

Name:____

ID#:____

S.No.#:____

Section#:____

Instructions:

1. Clearly indicate the theorem/result while applying it to solve a problem.
2. Indicate all calculations in the answer sheet.

Instructor: Dr. Qamrul Hasan Ansari

1. Find the equation of the tangent line to the curve $x = e^t$, $y = e^{-t}$ at $t = 1$ without eliminating the parameter. (4 marks)

OR

Find polar coordinates of all points at which the polar curve $r = a(1 + \cos\theta)$ has a vertical tangent. (4 marks)

Solution:

2. Find the area of the region inside the cardioid $r = 2 + 2\cos\theta$ and to the right of the line $r\cos\theta = \frac{3}{2}$. (5 marks)

Solution:

3. Find two unit vectors in 2-space that make an angle of 45° with $4\vec{i} + 3\vec{j}$. (5 marks)

Solution:

4. Find equation of the plane through the points $P(-2,1,4)$ and $Q(1,0,3)$ that is perpendicular to the plane $4x - y + 3z = 2$. (6 marks)

Solution:

5. Identify the surface $9x^2 + y^2 + 4z^2 - 18x + 2y + 16z = 10$ and make a rough sketch that shows its position, center and coordinates. (Do not give the orientation). (6 marks)

Solution:

6. Let $f(x, y) = xy \ln(x^2 + y^2)$. Is it possible to define $f(0, 0)$ so that f will be continuous at $(0, 0)$? (4 marks)

Solution:

7. The legs of a right triangle are measured to be 3 cm and 4 cm, with a maximum error of 0.05 cm in each measurement. Use differentials to approximate the maximum possible error in the calculated value of (a) the hypotenuse and (b) the area of the triangle. (8 marks)

OR

Two sides of a triangle have lengths $a = 5$cm and $b = 10$cm, and the included angle is $\theta = \pi/3$. If a is increasing at a rate of 2 cm/s, b is increasing at a rate of 1 cm/s, and θ remains constant, at what rate the third side changes? Is it increasing or decreasing? (8 marks)

Solution:

8. Find the directional derivative of $f(x, y) = e^{-x} \sec y$ at $P(0, \pi/4)$ in the direction of the origin.

(4 marks)

<u>Solution</u>:

9. Find an equation of the tangent plane to the ellipsoid $x^2 + 4y^2 + z^2 = 18$ at the point $(1, 2, 1)$ and determine the acute angle that this plane makes with XY-plane. (6 marks)

Solution:

10. A rectangular box with no top is to be constructed to have a volume $V = 12$ ft^3. The cost per square foot of the material to be used is SR. 4 for the bottom, SR. 3 for two of the opposite sides, and SR. 2 for remaining pair of opposite sides. Find the dimensions of the box that will minimize the cost. (8 marks)

Solution:

11. Use a double integral to find the volume of the solid in the first octant bounded above by $z = 9 - x^2$ below by $z = 0$, and laterally by $y^2 = 3x$. (6 marks)

OR

Evaluate $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dx\, dy$. (6 marks)

Solution:

12. Use a double integral to find the area of the region inside the circle $r = 1$ and outside the the cardioid $r = 1 + \cos\theta$. (6 marks)

Solution:

13. Let G be the wedge in the first octant cut from the cylindrical solid $y^2 + z^2 \leq 1$ by the planes $y = x$ and $x = 0$. Use a triple integral to find the volume of the wedge G. (6 marks)

Solution:

14. Use spherical coordinates to find the volume of the solid within the cone $\phi = \pi/4$ and between the spheres $\rho = 1$ and $\rho = 2$. (6 marks)

OR

Evaluate the following triple integral by converting into a spherical coordinates system. (6 marks)

$$\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\; dx\; dy.$$

Solution:

ROUGH WORK