King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

MATH - 201 Final Examination (051)

Venue: Bldg. 10
Time: 7:30 a.m. to 10:30 a.m.
Date: January 25, 2006
Max. Marks: 80

Name:_____

ID#:_____

S.No.#:_____

Section#:_____

Instructions:

1. Clearly indicate the theorem/result while applying it to solve a problem.
2. Indicate all calculations in the answer sheet.

Instructor: Dr. Qamrul Hasan Ansari

1. Find polar coordinates of all points at which the polar curve $r = a(1 + \cos\theta)$ has a vertical tangent line. (6 marks)

   Solution:

2. Sketch the curve $r^2 = \sin 2\theta$ and find the area inside this curve. (4 marks)

   Solution:

3. If $\overrightarrow{u} = \langle 3, 2, 1\rangle, \overrightarrow{v} = \langle 1, 1, 2\rangle$, and $\overrightarrow{w} = \langle 1, 3, 3\rangle$ are adjacent edges of a parallelepiped. Then find the angle between $\overrightarrow{u}$ and the plane containing the face determined by $\overrightarrow{v}$ and $\overrightarrow{w}$.

(6 marks)

Solution:

4. Find parametric equations of the line that is tangent to the parabola $y = x^2$ at the point $(-2, 4)$. (6 marks)

OR

Find the parametric equations of the line that contains the points $P(0, 2, 1)$ and intersects the line

$$L \quad : \quad x = 2t, \quad y = 1 - t, \quad z = 1 + t$$

at a right angle. (6 marks)

Solution:

5. Find an equation of a plane that contains the line $x = -2 + 3t$, $y = 4 + 2t$, and $z = 3 - t$ and is perpendicular to the plane $x - 2y + z = 5$. (6 marks)

OR

Find an equation of a plane that contains the point $P(2, 0, 3)$ and the line $x = -1 + t$, $y = t$, and $z = -4 + 2t$. (6 marks)

Solution:

6. If $x^2 + z \sin xyz = 0$, then calculate $\frac{\partial z}{\partial x}$ by using implicit differentiation. (4 marks)

Solution:

7. Suppose that a function $f(x, y)$ is differentiable at the point $(1, 1)$ with $f_x(1, 1) = 2$ and $f(1, 1) = 3$. Let $L(x, y)$ denote the local linear approximation of f at $(1, 1)$. If $L(1.1, 0.9) = 3.15$, find the value of $f_y(1, 1)$. (4 marks)

Solution:

8. Two sides of a triangle have lengths $a = 5$ cm and $b = 10$ cm, and the included angle is $\theta = \pi/3$. If a is increasing at a rate of 2 cm/s, b is increasing at a rate of 1 cm/s, and θ remains constant, at what rate the third side changes? (6 marks)

Solution:

9. Let $f(x, y) = x^2 e^y$. Find the maximum value of a directional derivative at $(-2, 0)$, and find the unit vector in the direction in which the maximum value occurs. (6 marks)

Solution:

10. Show that every line that is normal to the sphere $x^2 + y^2 + z^2 = 1$ passes through the origin. (6 marks)

OR

Find parametric equations for the tangent line to the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $x + 2y + 2z = 0$ at the point $(4, 3, 5)$. (6 marks)

Solution:

11. Find the absolute extrema of the function $f(x, y) = xy^2$ on the closed and bounded region R that satisfies the inequalities (8 marks)

$$x \geq 0, \quad y \geq 0, \quad \text{and} \quad x^2 + y^2 \leq 1.$$

OR

Find the point on the circle $x^2 + y^2 = 45$ that are closest to and farthest from $(1, 2)$.

(8 marks)

Solution:

12. Evaluate by reversing the order of integration (6 marks)

$$\int_0^2 \int_{y/2}^1 \cos(x^2) dx dy.$$

Solution:

13. Give the answer of any TWO questions:

(a) By using triple integral, find the volume of the solid enclosed between the paraboloids $z = 5x^2 + 5y^2$ and $z = 6 - 7x^2 - y^2$. (6 marks)

(b) Use cylindrical coordinates to evaluate the triple integral (6 marks)

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2 \, dz \, dy \, dx.$$

(c) Use spherical coordinates to evaluate the triple integral (6 marks)

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2+y^2+z^2} \, dz \, dy \, dx.$$

Solution:

ROUGH WORK