King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

MATH - 201 Final Examination (042)

Venue: Exb Center
Time: 7:00 p.m. to 10:00 p.m.
Date: June 08, 2005
Max. Marks: 80

Name:____

ID#:____

S.No.#:____

Section#:____

Instructions:

1. Clearly indicate the theorem/result while applying it to solve a problem.
2. Indicate all calculations in the answer sheet.

Instructor: Dr. Qamrul Hasan Ansari

1. Use the single integration method to find the area of the region enclosed by the inner loop of the limacon $r = 1 + 2\cos\theta$. (5 marks)

   Solution:

2. Find an equation of the plane passes through the point $(-1, 2, -5)$ and is perpendicular to the planes $2x - y + z = 1$ and $x + y - 2z = 3$. (5 marks)

   Solution:

3. Consider the parallelepiped with adjacent edges $\overrightarrow{u} = 3\overrightarrow{i} + 2\overrightarrow{j} + \overrightarrow{k}$, $\overrightarrow{v} = \overrightarrow{i} + \overrightarrow{j} + 2\overrightarrow{k}$, and $\overrightarrow{w} = \overrightarrow{i} + 3\overrightarrow{j} + 3\overrightarrow{k}$.
(a) Find the area of the faces determined by $\overrightarrow{u}$ and $\overrightarrow{w}$.
(b) Find the angle between $\overrightarrow{u}$ and the plane containing the faces determined by $\overrightarrow{v}$ and $\overrightarrow{w}$. (8 marks)

Solution:

4. Let L_1 and L_2 be the lines whose parametric equations are (8 marks)

$$L_1 \quad : \quad x = 4t, \quad y = 1 - 2t, \quad z = 2 + 2t$$

$$L_2 \quad : \quad x = 1 + t, \ y = 1 - t, \quad z = -1 + 4t.$$

(a) Find, to the nearest degree, the acute angle between L_1 and L_2 at their intersection.
(b) Find parametric equations for the line that is perpendicular to L_1 and L_2 and passes through their point of intersection.

Solution:

5. By taking the limit along different curves, show that the limit $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^4+z^4}$ does not exist. (6 marks)

OR

The area A of a triangle is given by $A = \frac{1}{2}ab\sin\theta$, where a and b are the lengths of two sides and θ is the angle between these sides. Suppose that $a = 5$, $b = 10$, and $\theta = \pi/3$.
(a) Find the rate at which A changes with respect to θ if a and b are held constant.
(b) Find the rate at which b changes with respect to a if A and θ are held constant.
(6 marks)

Solution:

6. The length and width of a rectangle are measured with errors of at most $r\%$, where r is small. Use differential to approximate the maximum percentage error in the calculated length of the diagonal. (7 marks)

OR

The legs of a right triangle are measured to be 3 cm and 4 cm, with a maximum error of 0.05 cm in each measurement. Use differentials to approximate the maximum possible error in the calculated value of (a) the hypotenuse and (b) the area of the triangle. (7 marks)

Solution:

7. Let $f(x, y) = \dfrac{y}{x+y}$. Find a unit vector $\overrightarrow{u}$ for which $D_{\overrightarrow{u}} f(2, 3) = 0$. (5 marks)

Solution:

8. Find parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point $(1, 1, 2)$. (8 marks)

Solution:

9. Find the absolute extrema of the function $f(x, y) = xy^2$ on the closed and bounded region R that satisfies the inequalities (8 marks)

$$x \geq 0, \quad y \geq 0, \quad \text{and} \quad x^2 + y^2 \leq 1.$$

OR

Find three positive numbers whose sum is 27 and such that the sum of their squares is as small as possible. (8 marks)

Solution:

10. Find the point on the plane $4x + 3y + z = 2$ that is closest to $(1, -1, 1)$. (8 marks)

Solution:

11. Give the answer of any TWO questions:

(a) Evaluate the double integral $\displaystyle\iint_R xy\,dA$; where R is the region enclosed by $y=\sqrt{x}$, $y = 6 - x$ and $y = 0$. (6 marks)

(b) By using triple integral, find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4$ and $z = 0$. (6 marks)

(c) Use spherical coordinates to evaluate the integral (6 marks)

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z^2\sqrt{x^2+y^2+z^2}\;dz\;dx\;dy.$$

Solution:

ROUGH WORK