King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

MATH - 201 Final Examination

Venue: MPH
Time: 7:30 a.m. to 10:30 a.m.
Date: January 06, 2005
Max. Marks: 80

Name:____

ID#:____

S.No.#:____

Section#:____

Instructions:

1. Clearly indicate the theorem/result while applying it to solve a problem.
2. Indicate all calculations in the answer sheet.

Instructor: Dr. Qamrul Hasan Ansari

1. Evaluate $\lim_{(x,y)\to(0,0)} y\ln(x^2+y^2)$. (3 marks)

OR

Do the points $(1,0,-1)$, $(0,2,3)$, $(-2,1,1)$ and $(4,2,3)$ lie in the same plane?

Solution:

2. Identify the surface $y^2 - \frac{x^2}{4} - \frac{z^2}{9} = 1$ and make a rough sketch that shows its position and orientation. (3 marks)

Solution:

3. Find the slope of the tangent line at $(-1, 1, 5)$ to the curve of intersection of the surface $z = x^2 + 4y^2$ and (3 marks)
(a) the plane $x = -1$ (b) the plane $y = 1$.

Solution:

4. Given that $e^{xy} \sinh z - z^2 x + 1 = 0$. Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ by using implicit differentiation. (3 marks)

Solution:

5. Find $\bigtriangledown w$ if $w = e^{-5x} \sec x^2 yz$. (3 marks)

Solution:

6. Find all values of t at which the parametric curve (5 marks)

$$x = 2t^3 - 15t^2 + 24t + 7, \qquad y = t^2 + t + 1$$

has (a) a horizontal tangent line and (b) a vertical tangent line.

Solution:

7. Use the single integration method to find the area of the region enclosed by the inner loop of the limacon $r = 1 + 2\cos\theta$. (5 marks)

   Solution:

8. Find, to the nearest degree, the angles that a diagonal of a box with dimensions 10 cm by 15 cm by 25 cm makes with the edges of the box. (5 marks)

   Solution:

9. Find the parametric equations of the line that is tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$. (5 marks)

OR

Show that the lines

$$L_1 \quad : \quad x = 2t, \quad y = 3 + 4t, \quad z = 2 - 6t$$

$$L_2 \quad : \quad x = 1 + 3t, \; y = 6t, \quad z = -9t$$

are parallel, and find the distance between them.

Solution:

10. Find parametric equations of the line through the point $(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$. (5 marks)

Solution:

11. Find a point on the surface $z = 8 - 3x^2 - 2y^2$ at which the tangent plane is perpendicular to the line $x = 2 - 3t$, $y = 7 + 8t$, $z = 5 - t$. (5 marks)

Solution:

12. Use a double integral in the polar coordinates to find the area of the region inside the circle $x^2 + y^2 = 4$ and to the right of the line $x = 1$. (5 marks)

Solution:

13. Evaluate the triple integral $\iiint\limits_G xyz dV$, where G is the solid in the first octant that is bounded by the parabolic cylinder $z = 2 - x^2$ and the planes $z = 0$, $y = x$ and $y = 0$. (5 marks)

Solution:

14. Use cylindrical coordinates to find the volume of the solid enclosed between the cone $z = (hr)/a$ and the plane $z = h$. (5 marks)

OR

Use spherical coordinates to evaluate the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}}\,dzdydx.$$

Solution:

15. The length, width, and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. (6 marks)

Solution:

16. Two sides of a triangle have lengths $a = 5$ cm and $b = 10$ cm, and the included angle is $\theta = \pi/3$. If a is increasing at a rate of 2 cm/s, b is increasing at a rate of 1 cm/s, and θ remains constant, at what rate is the third side changing? Is it increasing or decreasing? (6 marks)

Solution:

17. Find the absolute extrema of the function $f(x, y) = xe^y - x^2 - e^y$ on the closed and bounded rectangular region R with vertices $(0, 0)$, $(0, 1)$, $(2, 1)$ and $(2, 0)$. (8 marks)

OR

Find the extrema of the function $f(x, y, z) = xyz$ subject to $x^2 + y^2 + z^2 = 1$.

Solution: