DEANSHIP OF SCIENTIFIC RESEARCH

FAST TRACK RESEARCH GRANTS

FAST TRACK RESEARCH PROJECT NO. SF-2007/45

Title of the Proposal:

A Study of the Bivariate Unit-Gamma LogNormal (BUGL) Distribution

Duration of Project (in months)	**: 12**
Proposed Starting Date	**: January, 2007**
Ending Date	**: January, 2008**
Total Project Cost (SAR)	**: 49000**

Submitted by:

Signature: ……
Dr. Mohammad H. Omar, Assistant Professor, Math. Sciences

Signature: ……
Dr. Anwar H. Joarder, Associate Professor, Math. Sciences

Date: November 1, 2006

APPROVALS:……..……..

Chairman:…….. Date: ……
Department of Mathematical Sciences

Chairman,
Research Committee: ……. Date: ……..……

Vice Rector for Graduate Studies
and Scientific Research: ……..…….. Date:…….

عمادة البحث العلمي

مشروع بحث رقم :.....

عنوان البحث :

بحث الدراس على توزيع الثناء المتغيرات "وحدة–جامّا لوجنورمل (ب.و.ج.ل)"

مدة البحث : 12 شهر

تاريخ بدء البحث : يناير، 2007

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تكلفة المشروع (ريال) : 49000

مقدم من :

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التاريخ : 1 نوفمبر 2006

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ملخص

يهدف هذا المشروع الى البحث في بعض الخصائص المهمة لتوزيع ثنائي المتغيرات "وحدة–جامّا لوجنورمل" (unit-gamma LogNormal). لحد علمنا، لا يعرف التوزيع المشروط وحاصل ضرب العزوم فقط. لهذا التوزيع تطبيقات مجالات عدة، منها الطقس والتطبيقات المتعلقة بالبقاء (survival analysis).
في هذا البحث سنقوم باشتقاق بعض العزوم؛ العزم الثنائي وعزم Mahalanobis، ومعاملي الالتواء والتفرطح واقترانات مهمة اخرى.
كما ان هذا البحث سيساهم في ايجاد توزيع ثنائي المتغيرات جديد في عائلة توزيع "جامّا" (Gamma).

0. Abstract

The project aims at investigating some important characteristics of the unit-gamma LogNormal distribution. Only the conditional distribution and raw product moments are currently known. This bivariate distribution has found applications in areas such as survival data analysis and weather analysis. In this project, we will be deriving the central joint product moments, the standardized (Mahalanobis) moments, coefficients of skewness and kurtosis, the dependency function, Shannon entropy, hazard rate functions, reliability functions, and some other useful and related functions. Our investigation will lead to contribution of a relatively new bivariate distribution to the bivariate gamma family of distributions.

1. Introduction

Bivariate distributions of continuous, nonnegative random variables (X,Y), where Y is an additive component of X, occur in many real life situations, for example, survival data analysis, damaged observations, weather studies, et cetera. In such problems, the original random variable X may not be observable. However, if the conditional distribution of Y, given X, is assumed, then some of the distributional properties of the random variable X are tractable through the observed random variable Y. In this project, one such conditional distribution, namely, the unit-gamma is considered and the corresponding bivariate distribution of (X,Y) when X has the lognormal distribution is studied.

Properties of the bivariate normal distribution have been extensively studied in the literature. However, many applications in engineering and economics are concerned with distributions that are not necessarily normal. The bivariate unit-gamma lognormal (BUGL) distribution is one bivariate distribution that was introduced by Ratnaparkhi (1980) among others for situations where bivariate normality may not be applicable. Unfortunately, not much is presently known about the properties of this bivariate unit-gamma lognormal distribution.

2. Literature Review

In this project, we examine the bivariate unit-gamma lognormal distribution of continuous, nonnegative random variables (X,Y), where Y is an additive component of X. Such bivariate random variables occur in a natural way as

(1) 'failure' and 'warning' times (Mihram and Hulquist, 1967)

(2) 'true' and 'reported' incomes (Krishnaji, 1970)

(3) 'true' and 'stopped' (due to accident) life span of component in a system (Sethuraman, 1965).

However, generally a random variable Y alone is observable in practice. Therefore, the distributional consideration for the random variable (X,Y), for example, characterization, estimation et cetera are possible in the form of the conditional

distribution of *Y*, given *X*, is known. Mihram and Hulquist (1967) assumed that the conditional distribution of *Y*, given *X*, is Beta and studied the joint distribution of (*X,Y*), when *X* has the generalized gamma (Stacy) distribution. Instead of the generalized gamma distribution, Block and Raja Rao (1973) considered the generalized beta distribution of the second kind and studied the marginal distribution of *Y*.

The distribution referred to as "unit-gamma" in this project was considered by Grassia (1977) as a mixing distribution for the parameter 'p' of the binomial(n,p) distribution. It also appears as a 'log-gamma' distribution in the discussion of the splitting models (Schultz, 1975). Fields of applications where the lognormal family is useful, are recorded in Aitchison and Brown (1957) and Johnson and Kotz (1980). The generalized gamma distribution (Stacy, 1962) includes major 'life' distributions. Pareto law, besides it important role in economics, is also useful in the analysis of the survival data (Harris, 1968; Davis and Feldstein, 1979). A comprehensive literature review on this topic is needed for this project.

3. Project Objectives

The project aims at investigating some important characteristics of the unit-gamma LogNormal distribution. In this paper, we will be deriving

(a) some central joint product moments
(b) the standardized (Mahalanobis) moments
(c) coefficients of skewness and kurtosis
(d) the dependency function
(e) Shannon entropy function
(f) hazard rate functions
(g) reliability functions and
(h) some other useful and related functions.

We will also be developing different strategies for estimating parameters of this bivariate distribution. In addition, we will also provide surface analyses of the joint and marginal distributions and also correlation coefficient for certain values of the parameters of the distribution.

4. Description of the Problem and Proposed Method for Solution

Assuming that the distribution of *Y*, given *X*, is unit-gamma, we investigate the properties of the bivariate unit-gamma Lognormal distribution of (*X*,*Y*), when *X* has the Lognormal distribution.

The joint probability density function of the bivariate unit-gamma Lognormal (BUGL) distribution of (*X*,*Y*) is

$$f(x,y)=\frac{\alpha^{\beta}y^{\alpha-1}\left(-\log\frac{y}{x}\right)^{\beta-1}}{\Gamma(\beta)x^{\alpha}\sigma\sqrt{2\pi}}\exp[-\frac{1}{2\sigma^{2}}(\log x-\mu)^{2}]$$

where

$0 < y < x < \infty$; $-\infty < \mu < \infty$; $\sigma,\ \alpha,\ \beta > 0$.

The conditional probability density function of the unit-gamma distribution of *Y*, given *X* is

$$h(y\mid x)=\frac{\alpha^{\beta}y^{\alpha-1}\left(-\log\frac{y}{x}\right)^{\beta-1}}{\Gamma(\beta)x^{\alpha}}$$

where

$0 < y < x$; $\alpha,\ \beta > 0$

and the marginal probability distribution of *X* is lognormal with parameters μ and σ.

Ratnaparkhi (1980) derived the joint raw product moments of the bivariate unit-gamma Lognormal (BUGL) distribution of (*X*,*Y*) as follows:

$$E[X^{a}Y^{b}]=\frac{\alpha}{\alpha+b}E[X^{a+b}]$$

where

$$E[X^{a+b}]=\exp[(a+b)\mu+\frac{(a+b)^{2}}{2}\sigma^{2}],$$

$\alpha,\ \beta > 0$

a, b = 0, 1, 2, 3, ...

$\alpha+b\neq 0$

However, not much work has been done on the joint central product moments of the bivariate unit-gamma Lognormal (BUGL) distribution of (*X*,*Y*). In addition, many useful functions, such as the dependency function, Shannon entropy

function, hazard rate functions and reliability functions, appear to be under-explored.

Local dependence function (e.g. Wang, 1993), Shannon entropy function (e.g. Nadarajah and Zogfaros, 2005) and Reliability function (e.g. Nadarajah, 2005) will be analyzed. Some other type of dependency functions and hazard rate functions (e.g. Kotz, Balakrishnan and Johnson, 2000) will also be considered.

5. Work Plan for the Investigation

In the beginning of the project, we will make an extensive literature search, and collect relevant papers and books. The principal-investigator will start deriving the central moments by direct integration for the distribution. The co-investigator will assist the principal investigator to derive as many central moments as can be done under general conditions and will double-check the work of the principal investigator.

Next, the principal investigator will derive the standardized moments, coefficient of skewness and kurtosis, dependency function, Shannon entropy function, hazard rate functions, reliability functions and some other useful and related functions.

The co-investigator will develop estimation strategies for parameters of the BUGL distribution. He will also consolidate his works to write a report on the works done as well as a paper based on the findings of the investigation. The principal-investigator will consolidate all results to write a report on the joint work as well as a joint publication paper on the topic.

6. Personnel Requirements

1. Principal Investigator: The involvement of Dr. M.H.Omar will be 60% during one year.

2. Co-Investigator: The involvement of Dr. A. H. Joarder will be 40% during one year.

7. Usefulness of the Project

The work involved in the project is of the nature of basic research and its scientific merit is unlimited. The bivariate Unit-gamma LogNormal (BUGL) distribution is one active area of research with wide range of applications in

bivariate distribution theory and its applications. The results that would be obtained by the investigation would contribute to research activities in this area and stimulate further research.

8. Publication Plan

We expect to publish at least one research paper in a refereed international journal with possible title of "Some characteristics of the Bivariate Unit-gamma LogNormal (BUGL) Distribution".

9. References

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10. Budget (in SAR)

A. Principal Investigator:		
Dr. Mohammad H. Omar (ID # 7040504)		
Project Duration: 12 months		
Compensation per month (in SAR) @	1200	14400
A*. Conference Attendance by the **Principal Investigator**		8000
B. Co-Investigator:		
Dr. Anwar H. Joarder (ID # 6970819)		
Project Duration: 12 months		
Compensation per month (in SAR) @	1000	12000
C. Equipment (for the **Principal Investigator)**:		10500
High-Speed Laptop Computer	8000	
Laser Printer & Cartridge	2500	
D. Stationery and Copying		
Project Duration: 12 months		2000
E. Secretary		
Project Duration: 12 months		1500
F. Books and References		600
TOTAL (in SAR)		49000