**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**
**DEPARTMENT OF MATHEMATICAL SCIENCES**
**DHAHRAN, SAUDI ARABIA**

# STAT 211 (B): BUSINESS STATISTICS I

Semester 052
Major Exam #2
Wednesday April 26, 2006

Please **circle** your instructor′s name: Dr. Walid Abu-Dayyeh
Dr. Mohammad Omar
Mr. Marwan Al-Momani

Name: ID# Section: Serial:

| Question No | Full Marks | Marks Obtained |
|---|---|---|
| 1 | 10 | |
| 2 | 13 | |
| 3 | 10 | |
| 4 | 15 | |
| 5 | 5 | |
| 6 | 12 | |
| Total | 65 | |

**Question .1(*4+6=10-Points*)**

Ahmed followed stock exchanges over the past 50 days. In particular, he recorded price exchanges for two stocks, Al- Kahraba' and Safco. He partially constructed the following table.

| | | Safco price↓ | | | Total |
|---|---|---|---|---|---|
| | | Decrease | Unchanged | Increase | |
| Al-Kahraba ' price → | Decrease | 50 | 70 | *70* | 190 |
| | Unchanged | 10 | *50* | 40 | 100 |
| | Increase | 60 | 80 | *70* | *210* |
| **Total** | | 120 | 200 | 180 | 500 |

With this method, he partially completed the following table to study the behavior of the two stocks.

| | | Safco↓ | | | Total |
|---|---|---|---|---|---|
| | | Decrease | Unchanged | Increase | |
| Al-Kahraba ' → | Decrease | 0.10 | 0.14 | *0.14* | 0.38 |
| | Unchanged | 0.02 | *0.10* | 0.08 | 0.20 |
| | Increase | 0.12 | 0.16 | *0.14* | *0.42* |
| **Total** | | 0.24 | 0.40 | 0.36 | 1 |

With this partially constructed table, find the following:

a. What is the probability that Safco' stock price decreases given that Al-Kahraba' price decreases?

$P(Safco\ \ Price\ decrease \mid Al\text{-}Kahraba'\ decrease) = \frac{P(\text{both decrease})}{P(\text{Al-Kahraba' decrease})}$ $(1pt)$

$= \frac{0.10}{0.38}$

$= 0.263158$ $(1pt)$

b. Let A: Safco decreases and B: Al-Kahraba' stock decreases.

I. Are these two events mutually exclusive? Why?

No (1pt). $P(A\cap B)=0.10$

$P(A\cap B)\neq 0$. So, A and B are <u>NOT mutually exclusive</u> (1pt)

II. Are these two events independent? Why?

$P(A\cap B)=0.10$ No.(1pt)

$P(A)=0.38$ $P(A)P(B)=0.38(0.24)$

$P(B)=0.24$ $=0.0912$

$P(A\cap B)\neq P(A)P(B)$ (1pt). So A and B are <u>not</u> independent.

III. Find $P(A\cup B)$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$=0.38+0.24-0.10$ (1pt)

$=0.52$ (1pt)

**Question .2 (*1+2+4+2+4=13-Points*)**

The following distribution of number of daily customer complaints was observed for the past year at Giant Supermarket

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(X) | 0.15 | 0.30 | 0.20 | 0.15 | 0.14 | 0.06 |

a. What type of probability distribution is represented above?

Discrete Probability Distribution (1pt)

b. Find the probability that on a given day, there will be at most one customer complaint?

$P(x \le 1) = P(x=0) + P(x=1)$

$= 0.15 + 0.30$ (1pt)

$= 0.45$ (1pt)

c. Find the probability that on a given day, there will be between 2 and 4 complaints (inclusive) given that there is at most one complaint.

$$P(2 \le x \le 4 \mid x \le 3) = \frac{P(2 \le x \le 4 \text{ and } x \le 3)}{P(x \le 3)} \quad (1pt)$$

$$= \frac{P(2 \le x \le 3)}{1 - P(x > 3)} = \frac{P(x=2) + P(x=3)}{1 - (P(x=4) + P(x=5))}$$

$$= \frac{0.20 + 0.15}{1 - (0.14 + 0.06)} \quad (2pt)$$

$$= \frac{0.35}{0.80}$$

$$= 0.4375 \quad (1pt)$$

d. Find the expected number of customer complaints?

$$E[x] = \sum x P(x)$$

$= 0(0.15)+1(0.30)+2(0.20)+3(0.15)+4(0.14)+5(0.06)$ (1pt)

$= 2.01$ (1pt)

e. Find the standard deviation of customer complaints?

(1pt) This method

| X | P(X) | XP(X) | $X^2P(X)$ | or | $(X-E[X])^2P(X)$ |
|---|---|---|---|---|---|
| 0 | 0.15 | 0.00 | 0.00 | | 0.606015 |
| 1 | 0.30 | 0.30 | 0.30 | | 0.306030 |
| 2 | 0.20 | 0.40 | 0.80 | | 0.000020 |
| 3 | 0.15 | 0.45 | 1.35 | | 0.147015 |
| 4 | 0.14 | 0.56 | 2.24 | | 0.554414 |
| 5 | 0.06 | 0.30 | 1.50 | | 0.536406 |
| | 1.00 | 2.01 | 6.19 | | 2.1499 |
| | | | (1pt) | | (or 1pt) |

$$\sigma = \sqrt{\sum x^2 P(x) - (E[X])^2}$$

$$= \sqrt{6.19 - (2.01)^2}$$

$$= \sqrt{2.1499} \quad (1pt)$$

$$= 1.46625 \quad (1pt)$$

**Question .3(*3+4+3=10-Points*)**

The life time of batteries manufactured by a factory has an exponential distribution with mean 360 hours. A battery is selected randomly from the product of the factory. Then:

a. Find the probability that the battery will work at most 320 hours.

$$P(x \le 320) = P(0 \le x \le 320)$$

$$= 1 - e^{-\lambda(320)}$$ but what is $\lambda$? exponential mean $=1/\lambda$

360 hr $=1/\lambda$. So, $\lambda = 1/360$ (1 pt)

$$= 1 - e^{-320/360} \quad (1pt)$$

$$= 1 - e^{-0.8888889} \quad (1pt)$$

$$= 0.588889$$

b. Find the probability that the battery will work more than 420 hours given that it has worked more than 390 hours.

$$P(x > 420 \mid x > 390) = \frac{P(x > 420 \text{ and } x > 390)}{P(x > 390)} \quad (1pt)$$

$$= \frac{1-(1-e^{-\lambda 420})}{1-(1-e^{-\lambda 390})}$$

$$= \frac{e^{-420/360}}{e^{-390/360}} \quad (1pt)$$

$$= \frac{0.311403}{0.338465} \quad (1pt)$$

$$= 0.920044 \quad (1pt)$$

c. Find the median of the life time of the battery.

Exponential is skewed so mean is not median

$$P(0 \le x \le median) = 0.50 \quad (1pt)$$

$$1 - e^{-\lambda(median)} = 0.50$$

$$1 - e^{-median/360} = 0.50$$

$$e^{-median/360} = 0.50$$

$$\frac{-median}{360} = ln(0.50)$$

$$Median = -360 ln(0.50) \quad (1pt)$$

$$= 249.533 \quad (1pt)$$

**<u>Question .4(*3+4+4+4=15-Points*)</u>**

At KFUPM the distribution of student after-class daily studying time has been known to follow a ***normal distribution*** with a ***mean*** of ***110*** minutes and a ***standard deviation*** of ***36*** minutes.

a. A KFUPM student is randomly selected, what is the probability that he studies <u>less than</u> <u>126</u> minutes?

$P\left(x<126\right)=P\left(z<\frac{126-110}{36}\right)$

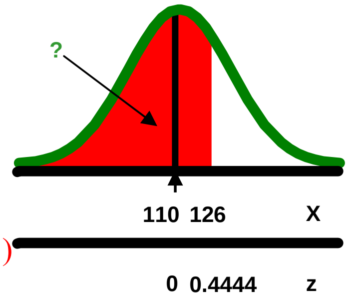
$=P\left(z<\frac{16}{36}\right)$

$=P(z<0.4444)$ $(1pt)$

$\simeq P(z<0)+P(0<z<0.44)$

$=0.5000+0.1700$ From the std Normal table $(1pt)$

$=0.6700$ $(1pt)$




b. If students who typically obtain A+ grades in their courses study <u>at least</u> 184 minutes daily, what is the percentage of these KFUPM students?

$P\left(x>184\right)=P\left(z>\frac{184-110}{36}\right)$ $(1pt)$

$=P(z>2.055556)\simeq P(z>2.06)$ $(1pt)$

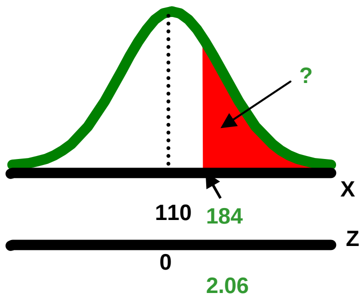
$=1-P(z>2.06)=1-[P(z<0)+P(0<z<2.06)]$

$=1-\left(0.5000+0.4803\right)$ From the std Normal table $(1pt)$

$=1-0.9803$

$=0.0197$ $(1pt)$

1.97% of KFUPM students




c. Find x where 88% of the students study less than x minutes.

$P(X<x)=0.8800$ $(1pt)$

$P(z<z_0)=0.8800$

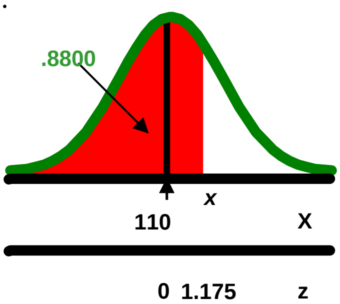
$P(0<z<z_0)=0.8800-0.5=0.3800$

BUT $z_0=1.175$ from std normal table

$z_0=\frac{x-\mu}{\sigma}=1.175$ $(1pt)$

$x=1.175(36)+110$ $(1pt)$

$=42.3+110=152.3$ $(1pt)$




d. If 8 KFUPM students are selected at random, then find the probability that at most one of them will study less than 126 minutes. From part a $p=0.6700$

binomial $n=8$ $p=0.6700$ $q=1-p=0.33$ $(1pt)$

$P\left(x\le 1\right)=P\left(x=0\right)+P(x=1)$ $(1pt)$

$=C_0^8p^0q^8+C_1^8p^1q^7$

$=(1)(1)(0.33)^8+\frac{8!}{1!7!}(0.67)(0.33)^7$ $(1pt)$

$=0.000141+8(0.67)(0.33)^7$

$=0.000141+0.002284$

$=0.002425$ $(1pt)$

**Question .5 (*3+2=5-Points*)**

The percentage of students who will be admitted to the university after taking an entrance exam is 66%. A random sample of 9 students from those who took the entrance exam is selected. Then:

a. Find the probability that 5 from them will be admitted to the university.

binomial $n = 9 \quad p = 0.66 \quad q = 1 - p = 0.34 \quad (1pt)$

$$P(x = 5) = C_5^9 p^5 q^4$$

$$= \tfrac{9!}{5!4!}(0.66)^5(0.34)^4 \qquad (1pt)$$

$$= \tfrac{9\times8\times7\times6}{4\times3\times2\times1}(0.66)^5(0.34)^4$$

$$= 0.210866 \qquad (1pt)$$

b. Find the expected number of students in the sample who will be admitted to the university.

$$E[x] = np = 9(0.66) \qquad (1pt)$$

$$= 5.94 \qquad (1pt)$$

**Question .6(*4+4+4=12-Points*)**

Suppose that on the average there are 5 car accidents weekly at the 3rd street. Then:

a. Find the probability that there will be at most 1 car accidents at the 3rd street next week.

Average= 5 cars/weekly $= \lambda$ and $t = 1$ So, $\lambda t = 5(1) = 5 \quad (1pt)$ Poisson

$$P(x \le 1) = P(x = 0) + P(x = 1) \qquad (1pt)$$

$$= \tfrac{(\lambda t)^0 e^{-\lambda t}}{0!} + \tfrac{(\lambda t)^1 e^{-\lambda t}}{1!}$$

$$= e^{-5} + 5e^{-5} \qquad (1pt)$$

$$= 6e^{-5}$$

$$= 0.040428 \qquad (1pt)$$

b. Find the probability that there will be at least 2 car accident at the 3rd street in the coming 2 weeks.

Average= 5 cars/weekly $= \lambda$ and $t = 2$ So, $\lambda t = 5(2) = 10 \quad (1pt)$ Poisson

$$P(x \ge 2) = 1 - [P(x = 0) + P(x + 1)]$$

$$= 1 - \left(\tfrac{(\lambda t)^0 e^{-\lambda t}}{0!} + \tfrac{(\lambda t)^1 e^{-\lambda t}}{1!}\right)$$

$$= 1 - (e^{-10} + 10e^{-10}) \qquad (1pt)$$

$$= 1 - 11e^{-10} = 1 - 0.000499 \qquad (1pt)$$

$$= 0.999501 \qquad (1pt)$$

c. Find the **mean** and **standard deviation** of the number of car accidents in two years. ( Hint: Use one year =53 weeks) $t = 2$ years $= 2(53)$ weeks

Mean

$$\mu = E[x] = \lambda t = 5(2)53) \qquad (1pt)$$

$$= 530 \qquad (1pt)$$

std deviation

$$\sigma = \sqrt{\lambda t}$$

$$= \sqrt{530} \qquad (1pt)$$

$$= 23.0217 \qquad (1pt)$$