$= \frac{1}{e}e|(z-(1-3i))| = |(z-(1-3i))| < 1$ Using $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

$\Rightarrow$ Radius of convergence is 1 and center of convergence is $1-3i$.

Q.9-(21.2): Determine radius of convergence of $\sum_{n=0}^{\infty}\left(\frac{e^{in}}{2n+1}\right)(z+4)^n$

$$R = \lim_{n\to\infty}\left|\left(\frac{e^{i(n+1)}}{2n+3}\right)(z+4)^{n+1}\frac{(2n+1)}{e^{in}}\frac{1}{(z+4)^n}\right| = \lim_{n\to\infty}\left|\frac{(2n+1)}{(2n+3)}e^{i}(z+4)\right| = |z+4| < 1$$

$\Rightarrow$ Radius of convergence is 1 and center of convergence is $-4+0i$.

Q.2-(21.3): $\sin(1-4i) = \sin 1\cosh 4 - i\cos 1\sinh 4$

Q.4-(21.3): $\tan(0+3i) = i\tanh 3$

Q.6-(21.3): $\cot\left(1-i\frac{\pi}{4}\right) = \dfrac{\cos\left(1-i\frac{\pi}{4}\right)}{\sin\left(1-i\frac{\pi}{4}\right)}$

$$\cos\left(1-i\frac{\pi}{4}\right) = \cos 1\cosh\tfrac{1}{4}\pi + i\sin 1\sinh\tfrac{1}{4}\pi$$

$$\sin\left(1-i\frac{\pi}{4}\right) = \sin 1\cosh\tfrac{1}{4}\pi - i\cos 1\sinh\tfrac{1}{4}\pi$$

$$\cot\left(1-i\frac{\pi}{4}\right) = \frac{\cos\left(1-i\frac{\pi}{4}\right)}{\sin\left(1-i\frac{\pi}{4}\right)} = \frac{\cos 1\cosh\frac{1}{4}\pi + i\sin 1\sinh\frac{1}{4}\pi}{\sin 1\cosh\frac{1}{4}\pi - i\cos 1\sinh\frac{1}{4}\pi}$$

$$= \frac{\left(\cos 1\cosh\frac{1}{4}\pi + i\sin 1\sinh\frac{1}{4}\pi\right)\left(\sin 1\cosh\frac{1}{4}\pi + i\cos 1\sinh\frac{1}{4}\pi\right)}{\left(\sin 1\cosh\frac{1}{4}\pi - i\cos 1\sinh\frac{1}{4}\pi\right)\left(\sin 1\cosh\frac{1}{4}\pi + i\cos 1\sinh\frac{1}{4}\pi\right)}$$

$$= \frac{i\sinh\frac{1}{4}\pi\cosh\frac{1}{4}\pi + \sin 1\cos 1}{\cosh^2\frac{1}{4}\pi - \cos^2 1}$$

Q.8-(21.3): $\cos(2-i) - \sin(2-i) = \cos 2\cosh 1 + i\sin 2\sinh 1 - \sin 2\cosh 1 + i\cos 2\sinh 1$

Q.11-(21.3): $z = x+iy,\ e^{z^2} = e^{x^2-y^2}e^{2(ixy)} = e^{x^2-y^2}(\cos(2xy) + i\sin(2xy))$

$= e^{x^2-y^2}\cos(2xy) + ie^{x^2-y^2}\sin(2xy)$

$u(x,y) = e^{x^2-y^2}\cos(2xy),\ v(x,y) = e^{x^2-y^2}\sin(2xy)$

$$\frac{\partial u(x,y)}{\partial x} = 2xe^{x^2-y^2}\cos 2xy - 2e^{x^2-y^2}(\sin 2xy)y = 2e^{x^2-y^2}[x\cos 2xy - y\sin 2xy]$$

$$\frac{\partial u(x,y)}{\partial y} = -2ye^{x^2-y^2}\cos 2xy - 2e^{x^2-y^2}(\sin 2xy)x = -2e^{x^2-y^2}[y\cos 2xy + x\sin 2xy]$$

$$\frac{\partial v(x,y)}{\partial x} = 2e^{x^2-y^2}(\sin 2xy)x + 2ye^{x^2-y^2}\cos 2xy = 2e^{x^2-y^2}[x\sin 2xy + y\cos 2xy]$$

$$\frac{\partial v(x,y)}{\partial y} = 2xe^{x^2-y^2}\cos 2xy - 2e^{x^2-y^2}(\sin 2xy)y = 2e^{x^2-y^2}[x\cos 2xy - y\sin 2xy]$$

CR equations are satisfied.

Q.15-(21.3): $\sin^2(z) = \sin^2 x\cosh^2 y + 2i\sin x\cosh y\cos x\sinh y - \cos^2 x\sinh^2 y$

$\cos^2(z) = \cos^2 x\cosh^2 y - 2i\sin x\cosh y\cos x\sinh y - \sin^2 x\sinh^2 y$

$\sin^2(z) + \cos^2(z) = \sin^2 x\cosh^2 y - \cos^2 x\sinh^2 y + \cos^2 x\cosh^2 y - \sin^2 x\sinh^2 y = 1$

Using $\sin^2(x) = 1 - \cos^2(x)$

Q.19-(21.3): $\sinh(z) = -i\sin(iz)$ and $\sin(w) = 0$ iff $w = n\pi$, n any integer.

Since $i \neq 0$, therefore $\sinh(z) = 0$ iff $iz = n\pi$ or $z = in\pi$, That is $y = n\pi$.