

## The *Cumulative Standard Normal Table*: Calculating Probabilities for the Standard Normal Distribution

To determine probabilities, we need to determine areas under the standard normal curve. We use the *Z-Table* to determine areas.

### □ *Z-Table*.

- The random variable  $z = \frac{x - m_x}{s_x}$  is a standard normal variable with mean of 0 and standard deviation of 1. (Any *x-value* can be transformed to a *z-value*.)
- The first column of the *Z-Table* lists the *z-value* to the first decimal place. This tells you which row to use.
- The first row of the *Z-Table* lists the second decimal place of the *z-value*. This tells you which column to use.
- These row and column values direct you to a value in the body of the table.

- **Procedure** – there are 4 cases that pretty much cover all the possible problems you'll face using the *Z-Table*. The following basic procedure should be followed to determine areas (or probabilities) using the cumulative Standard Normal Table:

**Step 1: Draw and illustrate the area you need. Practice – good drawings prevent errors!**

**Step 2: Approximate the area – take a guess about the probability you need.**

**Step 3: Locate the *z-value* or *z-values* in the *Z-Table* and the areas to the left of those values.** (Sometimes we need to use more than one *z-value*, and, remember, the *Cumulative Z-Table* always gives areas to the left.)

**Step 4: Use the area or areas to determine the probability you need. You may need to apply rules of probability in this step.**

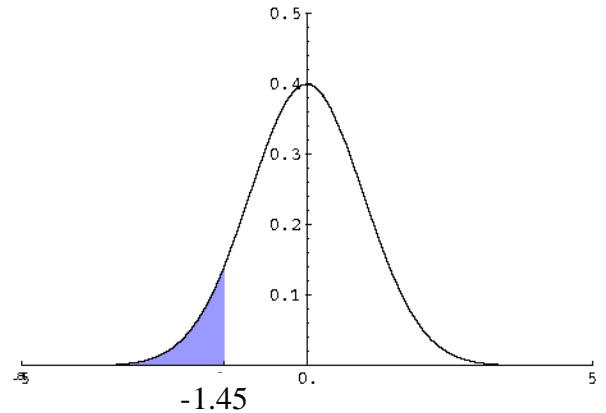
□ **Four Cases** – There are four cases to consider in determining probabilities or areas under the standard normal distribution. (Remember: probability = area.)

- **Case 1:** Determine the **area to the left of a  $z$ -value**. (This case is the easiest.)

**Example:** Given  $z = -1.45$ , determine  $P(z < -1.45)$ .

**Step 1:** Draw the standard normal and illustrate the area. (I used the computer to generate the image below, but you should practice drawing.)

**Step 2:** We know that the area must be less than 0.1587. Why? It looks like a fairly small area; I'm guessing this is an area of about 0.08. Around 8% of the distribution lies to the left of  $-1.45$ .



**Step 3:** Next, find the  $z$ -value  $-1.45$  in the table. Find  $-1.4$  in the first column of the table, move about to the center – the column labeled 0.05.

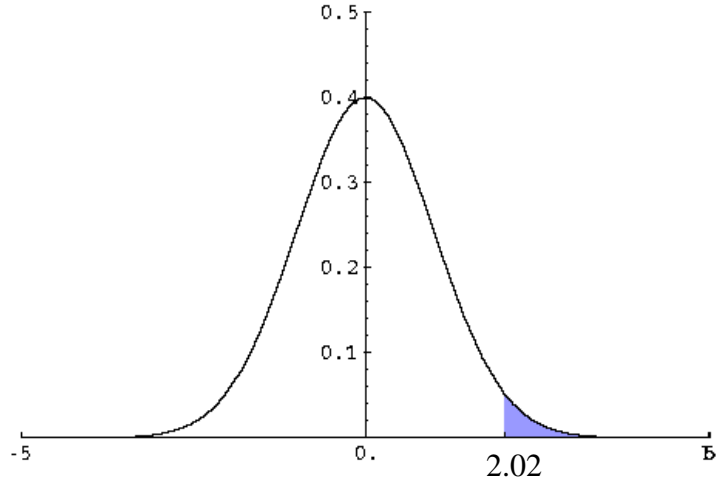
**Step 4:** The area given is 0.0735. Thus,  $P(z < -1.45) = 0.0735$ . These are the easiest problems to solve. The table always gives areas to the left.

- **Case 2:** Determine the **area to the right of the *z*-value**.

**Example:** Find the area to the **right** of  $z = 2.02$ . We need to determine  $P(Z > 2.02)$ .

**Step 1:** Draw and illustrate the area. The area we need is in the **right tail** this time.

**Step 2:** Approximate the area. This is a very small area; it must be about 0.05 or less. It's certainly much less than 0.5. We actually know that the area should be around 0.025. Why? Use the Empirical Rule.



**Step 3:** Locate the *z*-value:  $z = 2.02$  in the table.

**Step 4:** The area given is 0.9783. BUT, this is the area to the **left** of  $z = 2.02$ . And, we know from our drawing that the correct probability should be very small, around 0.025.) To determine the correct probability, apply the **Complement Rule** to calculate the area you need:

$$\text{Area to the right} = 1 - \text{Area to the left} = 1 - 0.9783 = 0.0217.$$

$$P(Z > 2.02) = 0.0217.$$

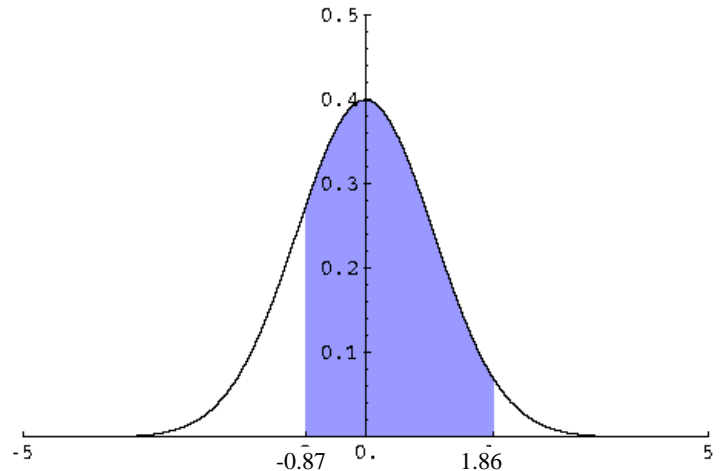
This is much closer to our guess of 0.025. Good drawings and good guesses help prevent mistakes.

- **Case 3:** Determine the area to the between two  $z$ -values.

$$P(a \leq Z \leq b) = \text{Area between points } a \text{ and } b.$$

**Example:** Find the area between the two values  $z = -0.87$  and  $z = 1.86$ .

**Step 1:** Draw the standard normal curve and illustrate the area you are to find. Here we locate the two values  $Z = -0.87$  and  $Z = 1.86$  on the graph and then shade the area between them.



**Step 2:** Approximate the area.

Take a guess. About how much of the distribution does this look like?

Approximately what area? We should use the Empirical Rule to help us. There should be an area of about 0.30 between  $z = -0.87$  and 0.00.

There should also be an area of about 0.45 or so from 0.00 to  $z = 1.86$ . Combined, we should expect an area of about 0.75.

**Step 3:** Find the  $z$ -value that is farthest to the right in the  $Z$ -Table:

Locate  $Z = 1.86$  in the table.  $P(Z < 1.86) = 0.9686$ .

Then, find the  $z$ -value that is farthest to the left in the  $Z$ -Table and determine that area:

Locate  $Z = -0.87$  in the table.  $P(Z < -0.87) = 0.1922$ .

From our drawing and guess, we know that the first probability is too big, the second is much too small.

**Step 4:** Determine the probability. We need to determine the probability that  $Z$  is **between two values**, which is equivalent to a **joint event**:  $P(Z < 1.86)$  and  $P(Z > -0.87)$ ; an intersection of two sets, or in this case, two areas.

Subtract the smaller area (area for the  $z$ -value farthest to the left) from the larger area (for the  $z$ -value that was farthest to the right):

$$P(-0.87 < Z < 1.86) = P(Z < 1.86) - P(Z < -0.87);$$

$$P(-0.87 < Z < 1.86) = 0.9686 - 0.1922 = 0.7764.$$

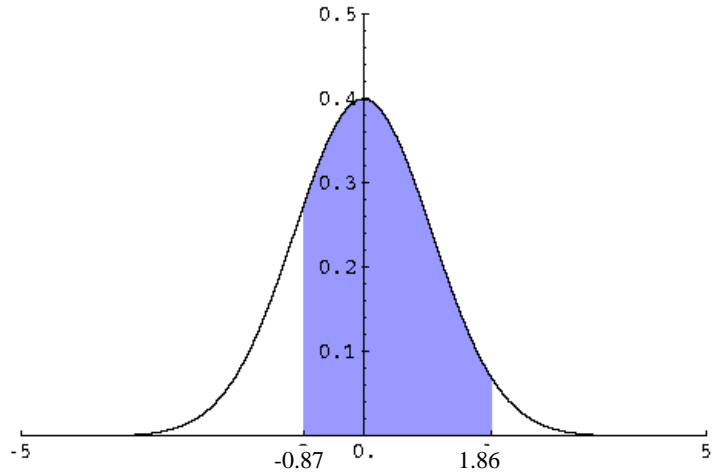
- **Case 4:** Determine the combined **area to the left of one z-value and to the right of another z-value**. (Here we combine Case 1 and Case 2 problems.)

$$P(Z \leq a \text{ or } Z > b)$$

**Example:** Find the area to the **left** of  $Z = -0.87$  and to the **right** of  $1.86$ .

**Step 1:** Draw and illustrate the area. This is the same as the previous example, **but now consider the unshaded areas in each tail.**

**Step 2:** Approximate the unshaded areas. I would guess its less than 0.5, probably 0.25 or less. (Not a lot of mystery here, we already know what the answer is.)



**Step 3:** Locate  $Z = -0.87$  in the table. The area to the **left** of  $Z = -0.87$  is:

$$P(Z \leq -0.87) = 0.1922.$$

Locate  $Z = 1.86$  in the *Z-Table*. The area in the table is the area to the **left** of  $Z = 1.86$ :

$$P(Z < 1.86) = 0.9686.$$

**Step 4:** From the *Z-Table*, we know:

$$P(Z \leq -0.87) = 0.1922, \text{ and } P(Z < 1.86) = 0.9686.$$

Apply the Complement Rule to find the **area to the right of  $Z = 1.86$** :

$$P(Z > 1.86) = 0.0314.$$

Next add these two areas (probabilities):

$$P(Z \leq -0.87 \text{ or } Z > 1.86) = 0.1922 + 0.0314 = 0.2236.$$

(There is no overlap or intersection to worry about here – just straightforward application of the Special Addition Rule.)

### More Examples – Symmetry.

- **Symmetry** – the standard normal curve is *symmetric* about its mean of 0. The standard deviation for the standard normal is 1.
- **Find Area between –1 and +1.** How might we rewrite this question?

*What is the area within 1 standard deviation of the mean?*

*or:*

*What is the probability of a z-value falling within 1 standard deviation of the mean?*

- This is Case 3:
  - Step 1: Draw and illustrate the area to be determined.
  - Step 2: Take a guess; it looks like more than 0.5.
  - Step 3: Find the value –1.00 in the *z-table*: this is the area to the left of –1.00.  
Area = 0.1587 ; or  $P(Z \leq -1) = 0.1587$ .  
Find the value +1.00 in the *z-table*: this is the area to the left of +1.00.  
Area = 0.8413 ; or  $P(Z \leq +1) = 0.8413$ .
  - Step 4: The area between the values –1 and +1 is:

$$P(-1 \leq Z \leq 1) = P(Z \leq 1) - P(Z \leq -1) = 0.8413 - 0.1587 = 0.6826.$$

- **Or: Use Symmetry – save a trip to the table.**

The area to the left of –1.00 was found to be 0.1587.

By symmetry, the area to the right of 1.00 must also be 0.1587. Then the combined area in the two tails is 0.3174.

By the **Complementation Rule**, the area between these two points must be:

$$P(-1 \leq Z \leq 1) = 1 - 0.3174 = 0.6826.$$

**Conclusion:** *68.26 % of the values for the standard normal lie within 1 standard deviation of the mean.*

- Use symmetry to verify that 50% of the distribution lies between –0.675 and +0.675.

The area to the left of –0.675 is about 0.25.

By symmetry, the area to the right of +0.675 is also 0.25. Then the combined area in the two tails to the left of –0.675 and to the right of +0.675 is 0.50.

By the **Complementation Rule**, the area between these two points is:

$$P(-0.675 \leq Z \leq +0.675) = 1 - 0.5000 = 0.5000.$$

- Case 4 examples – Use symmetry to verify that:
  - A combined 5% of the standard normal distribution lies to the left of  $-1.96$  and to the right of  $+1.96$ .
  - A combined 10% of the standard normal distribution lies to the left of  $-1.645$  and to the right of  $+1.645$ .
  - A combined 50% of the standard normal distribution lies to the left of  $-0.675$  and to the right of  $+0.675$ .
  
- Symmetry and the Standard Normal – What we know:
  - The standard normal is symmetric about the mean of 0. Then 50% of the values lie to the left of 0 and 50% of the values lie to the right of 0.
  - ***68.26 % of the values for the standard normal lie within 1 standard deviation of the mean, between  $-1$  and  $+1$ .***
  - ***95.44 % of the values for the standard normal lie between  $-2$  and  $+2$ .***
  - ***99.74 % of the values for the standard normal lie between  $-3$  and  $+3$ .***
  - ***50% of the values for the standard normal lie between  $-0.675$  and  $+0.675$ .***