

Solution Math 301-103      Sec: 03      Quiz 3 (A)

**Q.1:** Find the surface area of portion of the paraboloid  $2z = 3x^2 + 3y^2$  that is below the plane  $z = 3$ .

**Sol:** We need to evaluate  $\iint_R \sqrt{1 + (f_x)^2 + (f_y)^2} dA$ , where  $z = f(x, y) = \frac{3}{2}x^2 + \frac{3}{2}y^2$  and  $f_x(x, y) = 3x$ ,  $f_y(x, y) = 3y$ ,  $R$  is the region of the projection of the paraboloid onto  $xy$ -plane.

At  $z = 3$ ,  $2z = 3x^2 + 3y^2 \Rightarrow x^2 + y^2 = 2$ , a circle of radius  $\sqrt{2}$ . Therefore the projection of the paraboloid onto  $xy$ -plane is the circular disk  $x^2 + y^2 = 2$ .

$$A(S) = \iint_R \sqrt{1 + 4x^2 + 9y^2} dA = \int_0^{2\pi} \int_0^{\sqrt{2}} \sqrt{1 + 9r^2} r dr d\theta = \frac{2\pi}{27} ((19)^{3/2} - 1).$$

**Q.2:** Use the divergence theorem to evaluate  $\iint_S (\vec{F} \cdot \hat{n}) dS$  where  $\text{Div}(F) = \frac{1}{x^2 + y^2 + z^2}$  and  $D$  the region bounded by  $x^2 + y^2 + z^2 = 25$ ,  $x^2 + y^2 + z^2 = 16$ .

$$\begin{aligned} \text{Sol: } \iint_S (\vec{F} \cdot \hat{n}) dS &= \iiint_D \text{Div} F dV = \iiint_D \frac{1}{x^2 + y^2 + z^2} dV \\ &= \int_0^{2\pi} \int_0^\pi \int_4^5 \frac{1}{\rho^2} \rho^2 \sin(\phi) d\rho d\phi d\theta = (2\pi)(2)(5 - 4) = 4\pi. \end{aligned}$$

**Q.3:** Use Stokes's theorem to evaluate  $\oint_C \vec{F} \cdot d\vec{r}$ , where  $\vec{F} = 3z\hat{i} - 2x\hat{j} + 4y\hat{k}$  and  $S$  that portion of paraboloid  $z = 25 - x^2 - y^2$  for  $z \geq 0$ .

$$\text{Sol: } \text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3z & -2x & 4y \end{vmatrix} = 4\hat{i} + 3\hat{j} - 2\hat{k}.$$

$$g(x, y, z) = x^2 + y^2 + z - 25 = 0, \nabla g = \langle 2x, 2y, 1 \rangle \text{ and } \hat{n} = \frac{\langle 2x, 2y, 1 \rangle}{\sqrt{4x^2 + 4y^2 + 1}}$$

$$z = f(x, y) = 25 - x^2 - y^2 \Rightarrow f_x = -2x, f_y = -2y \text{ and } dS = \sqrt{1 + 4x^2 + 4y^2} dA$$

$$\begin{aligned} \oint_C \vec{F} \cdot d\vec{r} &= \iint_S \text{Curl } \vec{F} \cdot \hat{n} dS = \iint_R \frac{4x + 6y - 2}{\sqrt{1 + 4x^2 + 4y^2}} \sqrt{1 + 4x^2 + 4y^2} dA \\ &= \int_0^{2\pi} \int_0^5 [8r \cos(\theta) + 6r \sin(\theta) - 2] r dr d\theta = -50\pi \end{aligned}$$