

Name:.....Serial#:.....Sec #:.....

Q.1: Write the expression as a single logarithm with base 3 and simplify

$$3 + 3 \log_{\frac{1}{8}}(x^3 y^2) + 2 \log_{\sqrt[3]{4}} \sqrt[3]{x^4 y^4}$$

Sol: Using $\log_b b^x = x$ and $\log_{b^n} x = \frac{1}{n} \log_b x$, we get

$$\begin{aligned} 3 + 3 \log_{\frac{1}{8}}(x^3 y^2) + 2 \log_{\sqrt[3]{4}} \sqrt[3]{x^4 y^4} &= \log_2 8 + 3 \log_{2^{-3}}(x^3 y^2) + 2 \log_{2^{\frac{2}{3}}}(x^4 y^4)^{\frac{1}{3}} \\ &= \log_2 8 + 3 \cdot \frac{-1}{3} \log_2(x^3 y^2) + 2 \cdot \frac{3}{2} \cdot \frac{1}{3} \log_2(x^4 y^4) \\ &= \log_2 8 - \log_2(x^3 y^2) + \log_2(x^4 y^4) \\ &= \log_2 \left(\frac{8x^4 y^4}{x^3 y^2} \right) = \log_2(8xy^2). \end{aligned}$$

Q.2: Find the solution of the equation $\log_5(\log_2(\log_3 x^2)) = 0$ **Sol:** Using $\log_b x = y \Leftrightarrow b^y = x$,

$$\begin{aligned} \log_5(\log_2(\log_3 x^2)) = 0 &\Rightarrow \log_2(\log_3 x^2) = 5^0 = 1 \\ &\Rightarrow \log_3 x^2 = 2^1 = 2 \\ &\Rightarrow x^2 = 3^2 = 9 \\ &\Rightarrow x = \pm 3. \end{aligned}$$

Q.3: Write True or False:

$$(1) \log\left(4 + \frac{4}{2}\right) = \log(4) + \log\left(\frac{4}{3}\right) \quad \text{T}$$

$$(2) \log_b a > 0 \text{ if } 0 < a < 1 \text{ and } 0 < b < 1 \quad \text{T}$$

$$(3) \log_a x = \frac{1}{\log_b a + \log_x b} \quad \text{F}$$