

Name:.....Serial#:.....Sec #:.....

Q.1: Write the expression as a single logarithm with base 3 and simplify

$$2 + 3 \log_{\frac{1}{27}}(x^2 y^3) + 2 \log_{\sqrt[3]{9}} \sqrt{x^4 y^4}$$

Sol: Using $\log_b b^x = x$ and $\log_{b^n} x = \frac{1}{n} \log_b x$, we get

$$\begin{aligned} 2 + 3 \log_{\frac{1}{27}}(x^2 y^3) + 2 \log_{\sqrt[3]{9}} \sqrt{x^4 y^4} &= \log_3 9 + 3 \log_{3^{-3}}(x^2 y^3) + 2 \log_{3^{\frac{2}{3}}}(x^4 y^4)^{\frac{1}{2}} \\ &= \log_3 9 + 3 \cdot \frac{-1}{3} \log_3(x^2 y^3) + 2 \cdot \frac{3}{2} \cdot \frac{1}{3} \log_3(x^4 y^4) \\ &= \log_3 9 - \log_3(x^2 y^3) + \log_3(x^4 y^4) \\ &= \log_3 \left(\frac{9x^4 y^4}{x^2 y^3} \right) = \log_3(9x^2 y) . \end{aligned}$$

Q.2: Find the solution of the equation $\log_3(\log_2(\log_5 x^2)) = 0$ **Sol:** Using $\log_b x = y \Leftrightarrow b^y = x$,

$$\begin{aligned} \log_3(\log_2(\log_5 x^2)) = 0 &\Rightarrow \log_2(\log_5 x^2) = 3^0 = 1 \\ &\Rightarrow \log_5 x^2 = 2^1 = 2 \\ &\Rightarrow x^2 = 5^2 = 25 \\ &\Rightarrow x = \pm 5. \end{aligned}$$

Q.3: Write True or False:

$$(1) \log\left(3 + \frac{3}{2}\right) = \log(3) + \log\left(\frac{3}{2}\right) \quad \text{T}$$

$$(2) \log_b a > 0 \text{ if } a > 1 \text{ and } 0 < b < 1 \quad \text{F}$$

$$(3) \log_a x = \frac{1}{\log_b a \cdot \log_x b} \quad \text{T}$$