

Solution Math 302 – 02 Quiz 5

Name:.....Serial#:.....

Q.1: Evaluate $\int_{\Gamma} f(z)dz$, where $f(z) = 1 + z^2$, and Γ is a part of circle of radius 4 about the origin from $-4i$ to $4i$.

$$\Gamma : z(t) = 4e^{it} \text{ and } dz = 4ie^{it}, \text{ where } \frac{-\pi}{2} \leq t \leq \frac{\pi}{2}$$

$$\int_{\Gamma} f(z) dz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 16e^{2it}) 4ie^{it} dt = -\frac{104}{3}i$$

Q.2: Evaluate $\int_{\Gamma} f(z)dz$, where $f(z) = \bar{z}$, where Γ is the unit circle-about the origin.

$$\Gamma : z(t) = e^{it} \text{ and } dz = ie^{it}, \text{ where } 0 \leq t \leq 2\pi$$

$$\int_{\Gamma} f(z) dz = \int_0^{2\pi} e^{-it} ie^{it} dt = 2i\pi$$

Q.3: Evaluate $\int_{\Gamma} f(z)dz$, where $f(z) = \frac{z \sin(3z)}{(z+4)^3}$, Γ is a circle $|z - 2i| = 9$.

$$\begin{aligned} \int_{\Gamma} f(z)dz &= \int_{\Gamma} \frac{z \sin(3z)}{(z+4)^3} dz = \frac{2\pi i}{2!} \frac{d^2}{dz^2} [z \sin(3z)]_{z=-4} \\ &= \pi i \frac{d}{dz} [\sin(3z) + 3z \cos(3z)]_{z=-4} \\ &= \pi i \frac{d}{dz} [3 \cos(3z) + 3 \cos(3z) - 9z \sin(3z)]_{z=-4} \\ &= \pi i [6 \cos(12) - 36 \sin(12)] \end{aligned}$$