Solution Math 302 – 02 Quiz 3 (A)

Name:.....Serial#:.....

Q.1: Let $\mathbf{F}(t) = -(t + \sin(t))\,\mathbf{i} + \cos(2+t)\mathbf{j} + 3t^2\mathbf{k}$, and $f(t) = -2t^2$. Find $\mathbf{F}(f(t))$ and use this to compute $\dfrac{d}{dt}\mathbf{F}(f(t))$

Sol: $F(t) = [-t - \sin(t), \cos(2+t), 3t^2]$

$f(t) = -2t^2$

$F(f(t)) = [2t^2 + \sin 2t^2, \cos(-2 + 2t^2), 12t^4]$

$\frac{d}{dt}F(f(t)) = [4t + 4(\cos 2t^2)\,t, -4(\sin(-2+2t^2))\,t, 48t^3]$

Q.2: Let $\varphi(x, y, z) = e^x \sin(y) \sin(z)$. Find the gradient of φ and evaluate the gradient at $\left(0, \dfrac{\pi}{4}, -\dfrac{\pi}{4}\right)$. Also find the maximum and minimum rate of change φ at this point.

Sol: $\nabla\varphi(x, y, z) = (e^x \sin y \sin z, e^x \cos y \sin z, e^x \sin y \cos z)$

$\nabla\varphi(0, \frac{\pi}{4}, -\frac{\pi}{4}) = (\sin(\frac{\pi}{4})\sin(-\frac{\pi}{4}), \cos(\frac{\pi}{4})\sin(-\frac{\pi}{4}), \sin(\frac{\pi}{4})\cos(-\frac{\pi}{4})) = \left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

$Max = \left\|\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)\right\| = \frac{1}{2}\sqrt{3}$

$Min = -\left\|\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)\right\| = -\frac{1}{2}\sqrt{3}$

Q.3: Let $\mathbf{A} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ be a constant vector and $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Prove that divergence of $3\mathbf{A} - 5\mathbf{R}$ is equal to $\mathbf{-15}$.

Sol: $A = [a, b, c]$

$R = [x, y, z]$

$3A - 5R = [3a - 5x, 3b - 5y, 3c - 5z]$

$\nabla \cdot (3A - 5R) = -5 - 5 - 5 = -15$