

Solution Math 301-101 Sec: 02 & 03 Quiz 5

(B)

Q:1 Expand $f(x) = 1$, $0 < x < 2$ in a Fourier–Bessel series using Bessel functions of order zero that satisfy $3J_0(2\alpha) + 2\alpha J_0'(2\alpha)$. (Justify your answer with reason).

Sol: This is Case II with $h = 3, b = 2, n = 0$.

$f(x) = \sum_{i=1}^{\infty} c_i J_0(\alpha_i x)$, where

$$\begin{aligned} c_i &= \frac{2\alpha_i^2}{(\alpha_i^2 b^2 - n^2 + h^2) J_n^2(\alpha_i b)} \int_0^b x f(x) J_n(\alpha_i x) dx \\ &= \frac{2\alpha_i^2}{(4\alpha_i^2 + 9) J_0^2(2\alpha_i)} \int_0^2 x J_0(\alpha_i x) dx \\ &= \frac{2\alpha_i^2}{(4\alpha_i^2 + 9) J_0^2(2\alpha_i)} \int_0^{2\alpha_i} \frac{1}{\alpha_i^2} t J_0(t) dt, \text{ using } \alpha_i x = t \\ &= \frac{2}{(4\alpha_i^2 + 9) J_0^2(2\alpha_i)} \int_0^{2\alpha_i} \frac{d}{dt} [t J_1(t)] dt \\ &= \frac{2}{(4\alpha_i^2 + 9) J_0^2(2\alpha_i)} [2\alpha_i J_1(2\alpha_i)] = \frac{4\alpha_i J_1(2\alpha_i)}{(4\alpha_i^2 + 9) J_0^2(2\alpha_i)} \\ f(x) &= \sum_{i=1}^{\infty} \frac{4\alpha_i J_1(2\alpha_i)}{(4\alpha_i^2 + 9) J_0^2(2\alpha_i)} J_0(\alpha_i x). \end{aligned}$$

Q:2 Find first three nonzero terms if Fourier–Legendre expansion of the function

$$f(x) = \begin{cases} 0 & \text{if } -1 < x < 0 \\ 5x^2 & \text{if } 0 \leq x < 1 \end{cases}.$$

Sol: $f(x) = \sum_{n=1}^{\infty} c_n P_n(x)$, where,

$$c_n = \frac{2n+1}{2} \int_{-1}^1 f(x) P_n(x) dx, n = 0, 1, 2, \dots, P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$c_0 = \frac{1}{2} \int_{-1}^1 5x^2 dx = \frac{5}{6}$$

$$c_1 = \frac{3}{2} \int_{-1}^1 5x^3 dx = \frac{15}{8}$$

$$c_2 = \frac{5}{2} \int_{-1}^1 5x^2 \frac{1}{2}(3x^2 - 1) dx = \frac{25}{4} \int_{-1}^1 (3x^4 - x^2) dx = \frac{5}{3}$$

$$f(x) = \frac{5}{6} P_0(x) + \frac{15}{8} P_1(x) + \frac{5}{3} P_2(x)$$