Math 301-122 Quiz 5 (A)

Name:.....Sec#:.....ID#:.....Ser#:.....

Q.1: Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ subject to the conditions $u(0,t) = 0,\ u(\pi,t) = 0$ and $u(x,0) = 0,\ u_t(x,0) = 5\sin(x) + 2\sin(3x)$.

$$\frac{d^2U}{dx^2} = s^2U - s\,u(x,0) - u_t(x,0)$$

$$\frac{d^2U}{dx^2} - s^2U = -5\sin x - 2\sin 3x$$

To find U_c: $\frac{d^2U}{dx^2} - s^2U = 0$ $\qquad m^2 - s^2 = 0$, $m = \pm s$

$$U_c = A\cosh sx + B\sinh sx$$

$u(0,t) = 0 \Rightarrow U(0,s) = 0 \Rightarrow A = 0$

$u(\pi,t) = 0 \Rightarrow U(\pi,s) = 0 \Rightarrow B\sinh \pi s = 0 \Rightarrow B = 0$

So $U_c = 0$

To find U_p: Let $U_p = A\sin x + B\sin 3x$

Then $U_p'' = -A\sin x - 9B\sin 3x$

and $-A\sin x - 9B\sin 3x - s^2A\sin x - s^2B\sin 3x$

$= -5\sin x - 2\sin 3x$

$\Rightarrow A = \frac{5}{s^2+1}$, $B = \frac{2}{s^2+9}$

$$U = \frac{5}{s^2+1}\sin x + \frac{2}{3}\frac{3}{s^2+9}\sin 3x$$

$$u(x,t) = 5\sin t\sin x + \frac{2}{3}\sin 3t\sin 3x$$

Math 301-122 Quiz 5 (B)

Name:.....Sec#:.....ID#:.....Ser#:.....

Q.1: Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ subject to the conditions $u(0,t) = 0$, $u(1,t) = 0$ and $u(x,0) = 3\sin(2\pi x) + 5\sin(3\pi x)$, $u_t(x,0) = 0$.

$$\frac{d^2U}{dx^2} = s^2U - su(x,0) - u_t(x,0)$$

$$\frac{d^2U}{dx^2} - s^2U = -3s\sin(2\pi x) - 5s\sin(3\pi x)$$

To find U_c: $\frac{d^2U}{dx^2} - s^2U = 0$, $m^2 - s^2 = 0$, $m = \pm s$

$$U_c = A\cosh sx + B\sinh sx$$

$u(0,t) = 0 \Rightarrow U(0,s) = 0 \Rightarrow A = 0$

$u(1,t) = 0 \Rightarrow U(1,s) = 0 \Rightarrow B\sinh s = 0$

$\Rightarrow B = 0$, $s > 0$

So $U_c = 0$

To find U_p: let $U_p = A\sin(2\pi x) + B\sin(3\pi x)$

Then $U_p'' = -4\pi^2 A\sin(2\pi x) - 9\pi^2 B\sin(3\pi x)$

and $-4\pi^2 A\sin(2\pi x) - 9\pi^2 B\sin(3\pi x)$

$- s^2 A\sin(2\pi x) - s^2 B\sin(3\pi x)$

$= -3s\sin(2\pi x) - 5s\sin(3\pi x)$

$\Rightarrow A = \frac{3s}{s^2+4\pi^2}$, $B = \frac{5s}{s^2+9\pi^2}$

$$U(x,s) = \frac{3s}{s^2+4\pi^2}\sin(2\pi x) + \frac{5s}{s^2+9\pi^2}\sin(3\pi x) \Rightarrow$$

$u(x,t) = 3\cos 2\pi t \sin 2\pi x + 5\cos 3\pi t \sin 3\pi x$