Solution Math 301-101 Sec: 02 & 03 Quiz 5 (A)

Q:1 Expand $f(x) = 1$, $0 < x < 3$ in a Fourier–Bessel series using Bessel functions of order zero that satisfy $2J_0(3\alpha) + 3\alpha J_0'(3\alpha)$. (Justify your answer with reason).

Sol: This is Case II with $h = 2, b = 3, n = 0$.

$f(x) = \sum_{i=1}^{\infty} c_i J_0(\alpha_i x)$, where

$$c_i = \frac{2\alpha_i^2}{(\alpha_i^2 b^2 - n^2 + h^2)J_n^2(\alpha_i b)} \int_0^b x f(x) J_n(\alpha_i x) dx$$
$$= \frac{2\alpha_i^2}{(9\alpha_i^2 + 4)J_0^2(3\alpha_i)} \int_0^3 x J_0(\alpha_i x) dx$$
$$= \frac{2\alpha_i^2}{(9\alpha_i^2 + 4)J_0^2(3\alpha_i)} \int_0^{3\alpha_i} \frac{1}{\alpha_i^2} t J_0(t) dt, \text{ using } \alpha_i x = t$$
$$= \frac{2}{(9\alpha_i^2 + 4)J_0^2(3\alpha_i)} \int_0^{3\alpha_i} \frac{d}{dt}[tJ_1(t)] dt$$
$$= \frac{2}{(9\alpha_i^2 + 4)J_0^2(3\alpha_i)} [3\alpha_i J_1(3\alpha_i)] = \frac{6\alpha_i J_1(3\alpha_i)}{(9\alpha_i^2 + 4)J_0^2(3\alpha_i)}$$
$$f(x) = \sum_{i=1}^{\infty} \frac{6\alpha_i J_1(3\alpha_i)}{(9\alpha_i^2 + 4)J_0^2(3\alpha_i)} J_0(\alpha_i x).$$

Q:2 Find first three nonzero terms if Fourier–Legendre expansion of the function

$$f(x) = \begin{cases} 0 & \text{if } -1 < x < 0 \\ 3x^2 & \text{if } 0 \le x < 1 \end{cases}.$$

Sol: $f(x) = \sum_{n=1}^{\infty} c_n P_n(x)$, where,

$c_n = \frac{2n+1}{2} \int_{-1}^{1} f(x) P_n(x) dx, n = 0, 1, 2, ..., P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1)$

$c_0 = \frac{1}{2} \int_{-1}^{1} 3x^2 dx = \dfrac{1}{2}$

$c_1 = \frac{3}{2} \int_{-1}^{1} 3x^3 dx = \dfrac{9}{8}$

$c_2 = \frac{5}{2} \int_{-1}^{1} 3x^2 \frac{1}{2}(3x^2 - 1) dx = \frac{15}{4} \int_{-1}^{1} (3x^4 - x^2) dx = 1$

$f(x) = \frac{1}{2} P_0(x) + \frac{9}{8} P_1(x) + P_2(x)$