Math 301-103 Sec: 01 Quiz 4 (A)

Q.1: Solve $y'' + 5y' + 4y = 0$ with $y(0) = 1$ and $y'(0) = 0$.

Sol: $s^2Y(s) - sy(0) - y'(0) + 5sY(s) - 5y(0) + 4Y(s) = 0$

$$\Rightarrow (s^2 + 5s + 4)Y(s) = s + 5$$

$$\Rightarrow Y(s) = \frac{s+5}{s^2+5s+4} = \frac{4/3}{s+1} - \frac{1/3}{s+4}$$

$$\Rightarrow y(t) = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}.$$

Q.2: Find $\mathcal{L}^{-1}\left\{\dfrac{(2s+1)e^{-2s}}{s^2+4}\right\}$.

Sol: $$\mathcal{L}^{-1}\left\{\frac{(2s+1)e^{-2s}}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{2se^{-2s}}{s^2+4} + \frac{e^{-2s}}{s^2+4}\right\}$$
$$= \left(2\cos 2(t-2) + \frac{1}{2}\sin 2(t-2)\right)\mathcal{U}(t-2)$$

Q.3: Solve $f(t) = 2 - 4\int_0^t \sin(\tau)f(t-\tau)d\tau$ for $f(t)$.

Sol: $F(s) = \dfrac{2}{s} - \dfrac{4}{s^2+1}F(s) \Rightarrow \left(\dfrac{s^2+5}{s^2+1}\right)F(s) = \dfrac{2}{s} \Rightarrow F(s) = \dfrac{2s^2+2}{s(s^2+5)}$

$$F(s) = \frac{2/5}{s} + \frac{8/5s}{s^2+5} \Rightarrow f(t) = \frac{2}{5} + \frac{8}{5}\cos(\sqrt{5}t)$$