Solution Math 301-101 Sec: 02 & 03 Quiz 3 (B)

Q 1: Find $\mathcal{L}^{-1}\left\{\dfrac{2s-1}{(s^2+s)(s^2+1)}\right\}$.

Sol: Using partial fractions, we get

$$\frac{2s-1}{(s^2+s)(s^2+1)}=\frac{2s-1}{s(s+1)(s^2+1)}=\frac{-1}{s}+\frac{\frac{3}{2}}{s+1}+\frac{\frac{-1}{2}s+\frac{3}{2}}{s^2+1}=\frac{-1}{s}+\frac{\frac{3}{2}}{s+1}+\frac{\frac{-1}{2}s}{s^2+1}+\frac{\frac{3}{2}}{s^2+1}$$

$$\mathcal{L}^{-1}\left\{\frac{-1}{s}+\frac{\frac{3}{2}}{s+1}+\frac{\frac{-1}{2}s}{s^2+1}+\frac{\frac{3}{2}}{s^2+1}\right\}=-1+\tfrac{3}{2}e^{-t}-\tfrac{1}{2}\cos(t)+\tfrac{3}{2}\sin(t)$$

Q 2: Use laplace transform to solve $y''-4y'+8y=0,\ y(0)=0,\ \ y'(0)=-5.$

Sol: Using partial fractions, we get

$$s^2Y(s)-sy(0)-y'(0)-4sY(s)+4y(0)+8Y(s)=0$$

$$(s^2-4s+8)Y(s)=-5\Rightarrow Y(s)=\frac{-5}{s^2-4s+8}=\frac{-5}{(s-2)^2+2^2}$$

$$y(t)=-\tfrac{5}{2}e^{2t}\sin(2t).$$

Q 3: Find $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{(s^2+4)}\right\}$ and find $\mathcal{L}\left\{\sin(t)u\left(t-\frac{\pi}{2}\right)\right\}$.

Sol: $\dfrac{1}{2}\mathcal{L}^{-1}\left\{\dfrac{2}{(s^2+4)}e^{-2s}\right\}=\dfrac{1}{2}\sin 2(t-2)u(t-2)$

$$\mathcal{L}\left\{\sin(t)u\left(t-\tfrac{\pi}{2}\right)\right\}=e^{-\frac{\pi}{2}s}\mathcal{L}\left\{\sin(t+\tfrac{\pi}{2})\right\}=e^{-\frac{\pi}{2}s}\mathcal{L}\left\{\cos(t)\right\}=e^{-\frac{\pi}{2}s}\frac{s}{s^2+1}.$$

Q 4: Solve the Volterra Integral equation $f(t)=\cos(t)+\int_0^t e^{-\tau}f(t-\tau)\,d\tau$.

Sol: $f(t)=\cos(t)+\int_0^t e^{-\tau}f(t-\tau)\,d\tau=\cos(t)+e^{-t}*f(t)$

Taking Laplace transform we get

$$F(s)=\frac{s}{s^2+1}+\frac{1}{s+1}F(s)$$

$$\Rightarrow\left(1-\frac{1}{s+1}\right)F(s)=\frac{s}{s^2+1}$$

$$\Rightarrow F(s)=\frac{s+1}{s^2+1}=\frac{s}{s^2+1}+\frac{1}{s^2+1}$$

$$\Rightarrow f(t)=\cos(t)+\sin(t).$$