Math 301-122 Quiz 3 (A)

Name:..Sec#:..................ID#:.................................Ser#:.........

Q.1: Find $\mathcal{L}\{t^2\cos(t)\}$

$$\mathcal{L}\{t^2 \cos t\} = (-1)^2 \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$= \frac{d}{ds}\left[\frac{s^2+1-s\cdot 2s}{(s^2+1)^2}\right] = \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$$

$$= \frac{-2s(s^2+1)^2-(1-s^2)\cdot 2(s^2+1)\cdot 2s}{(s^2+1)^4} = \frac{-2s(s^2+1)-4s(1-s}{(s^2+1)^3}$$

$$= \frac{-2s^3-2s-4s+4s^3}{(s^2+1)^3} = \frac{2s^3-6s}{(s^2+1)^3}.$$

Q.2: Solve the integral equation $f(t) = \cos(t) + \int_0^t e^{-\tau} f(t-\tau)d\tau$

$$F(s) = \frac{s}{s^2+1} + \frac{1}{s+1}F(s)$$

$$\left(1-\frac{1}{s+1}\right)F(s) = \frac{s}{s^2+1}$$

$$\frac{s}{s+1}F(s) = \frac{s}{s^2+1}$$

$$F(s) = \frac{s+1}{s^2+1} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$$f(t) = \cos t + \sin t$$

3(A)

Q.3: Solve the differential equation $y'' + y = \delta(t - \frac{\pi}{2}) + \delta(t - \frac{3\pi}{2})$, $y(0) = 0$, $y'(0) = 0$

$$s^2 Y(s) - s y(0) - y'(0) + Y(s) = e^{-\frac{\pi}{2}s} + e^{-\frac{3\pi}{2}s}$$

$$(s^2+1) Y(s) = e^{-\frac{\pi}{2}s} + e^{-\frac{3\pi}{2}s}$$

$$Y(s) = e^{-\frac{\pi}{2}s} \frac{1}{s^2+1} + e^{-\frac{3\pi}{2}s} \frac{1}{s^2+1}$$

$$y(t) = \sin(t - \tfrac{\pi}{2}) \mathcal{U}(t - \tfrac{\pi}{2})$$

$$+ \sin(t - \tfrac{3\pi}{2}) \mathcal{U}(t - \tfrac{3\pi}{2})$$

$$= -\cos t \, \mathcal{U}(t - \tfrac{\pi}{2}) + \cos t \, \mathcal{U}(t - \tfrac{3\pi}{2})$$

Math 301-122 Quiz 3 (B)

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Q.1: Find $\mathcal{L}\{3t^2\sin(t)\}$

$$\mathcal{L}\{3t^2 \sin t\} = 3\frac{d^2}{ds^2}\left[\frac{1}{s^2+1}\right]$$

$$= 3\frac{d}{ds}\left[\frac{-2s}{(s^2+1)^2}\right] = 3\left[\frac{-2(s^2+1)^2 + 2s\cdot 2(s^2+1)\cdot 2s}{(s^2+1)^4}\right]$$

$$= 3\,\frac{-2(s^2+1)+8s^2}{(s^2+1)^3} = \frac{3(6s^2-2)}{(s^2+1)^3} = \frac{6(3s^2-1)}{(s^2+1)^3}$$

Q.2: Solve the integral equation $f(t) = 3t^2 - e^{-t} - \int_0^t e^{t-\tau} f(\tau)d\tau$

$$F(s) = \frac{3\cdot 2}{s^3} - \frac{1}{s+1} - \frac{1}{s-1}F(s)$$

$$\left[1+\frac{1}{s-1}\right]F(s) = \frac{6}{s^3} - \frac{1}{s+1}$$

$$\frac{s}{s-1}F(s) = \frac{6}{s^3} - \frac{1}{s+1}$$

$$F(s) = \frac{6(s-1)}{s^4} - \frac{s-1}{s(s+1)}$$ Partial Fraction

$$= \frac{6}{s^3} - \frac{6}{s^4} + \frac{1}{s} - \frac{2}{s+1}$$

$$f(t) = 3t^2 - t^3 + 1 - 2e^{-t}$$

Q.3: Solve the differential equation $y'' + y = \delta(t-2\pi) + \delta(t-4\pi)$, $y(0) = 1$, $y'(0) = 0$

$$s^2 Y(s) - s\,y(0) - \cancel{y'(0)} + Y(s) = e^{-2\pi s} + e^{-4\pi s}$$

$$(s^2+1)\,Y(s) = s + e^{-2\pi s} + e^{-4\pi s}$$

$$Y(s) = \frac{s}{s^2+1} + e^{-2\pi s}\frac{1}{s^2+1} + e^{-4\pi s}\frac{1}{s^2+1}$$

$$y(t) = \cos t + \sin(t-2\pi)\,\mathcal{U}(t-2\pi) + \sin(t-4\pi)\,\mathcal{U}(t-4\pi)$$

$$= \cos t + \sin t\,\mathcal{U}(t-2\pi) + \sin t\,\mathcal{U}(t-4\pi)$$